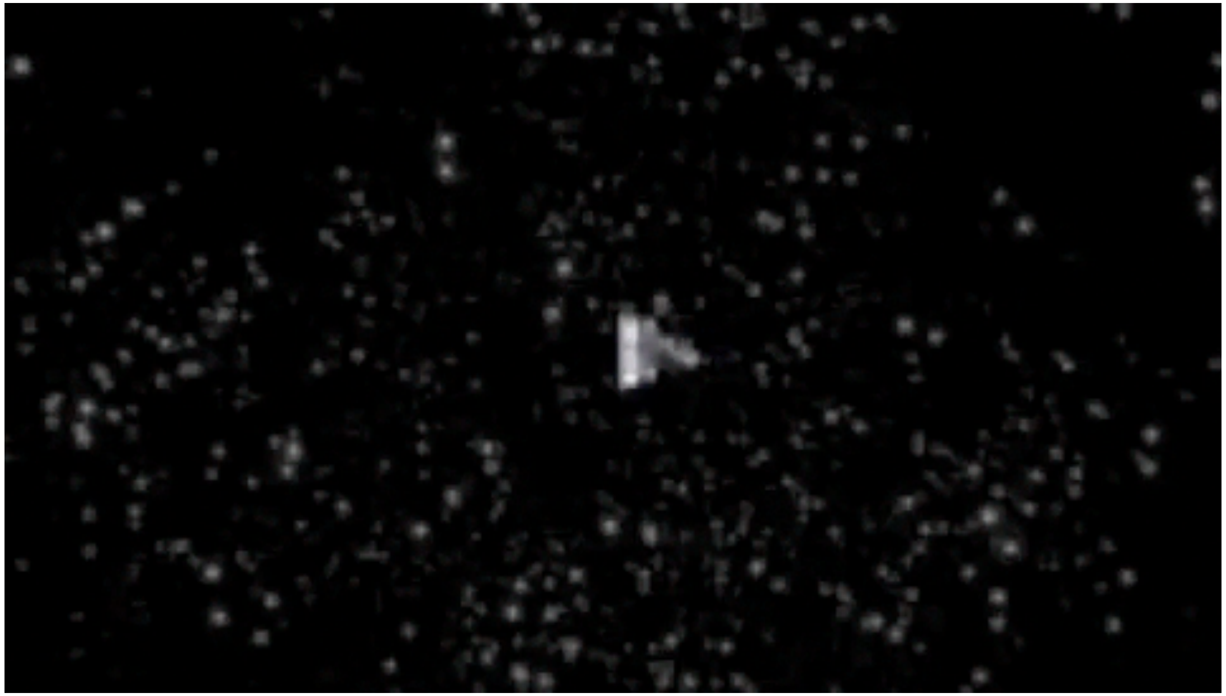

Cosmic Microwave Background Map Reconstruction
Launching of Planck
Ariane 5 : May, 14, 2009
J.-L. Starck, J. Bobin, F. Sureau
Planck Mission Duration : 2.5 years
Until end of 2011

The Cosmic Microwave Background

- The Universe is filled with a blackbody radiation field at a temperature of 3K.
- Predicted by G. Gamow in 1948
- Observed for the first time by Penzias and Wilson (1965)
- Confirmed by COBE (1990)
- Spectacular measurement of anisotropies by WMAP
- WMAP observed the CMB since 2002. Fifth and Last data release in August 2011.
- PLANCK first cosmological results in January 2013.

PLANCK and Sparsity



•Successor of WMAP (better resolution, better sensitivity, more channels)

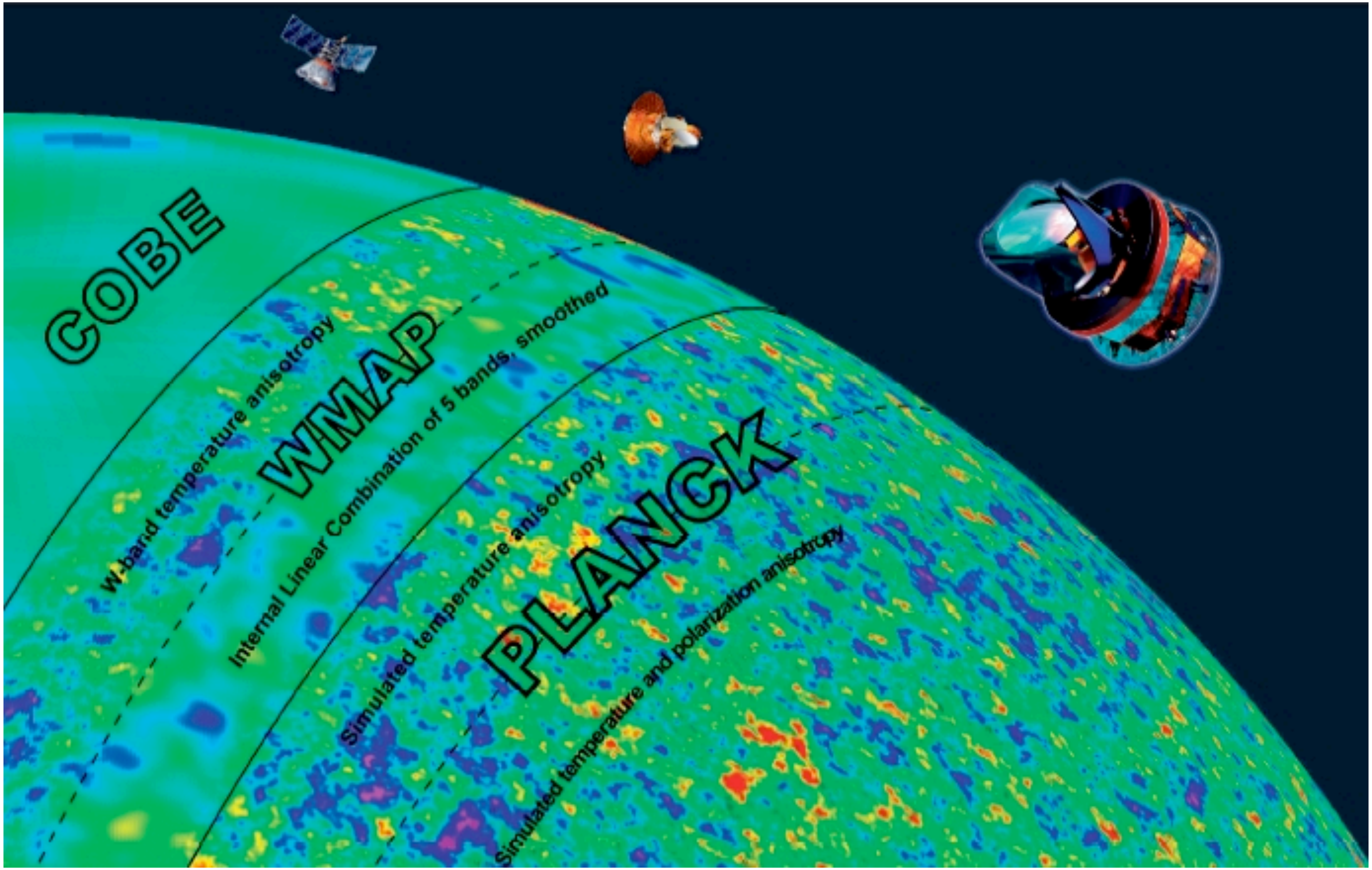
•Launched on May 14, 2009

•Two instruments LFI and HFI

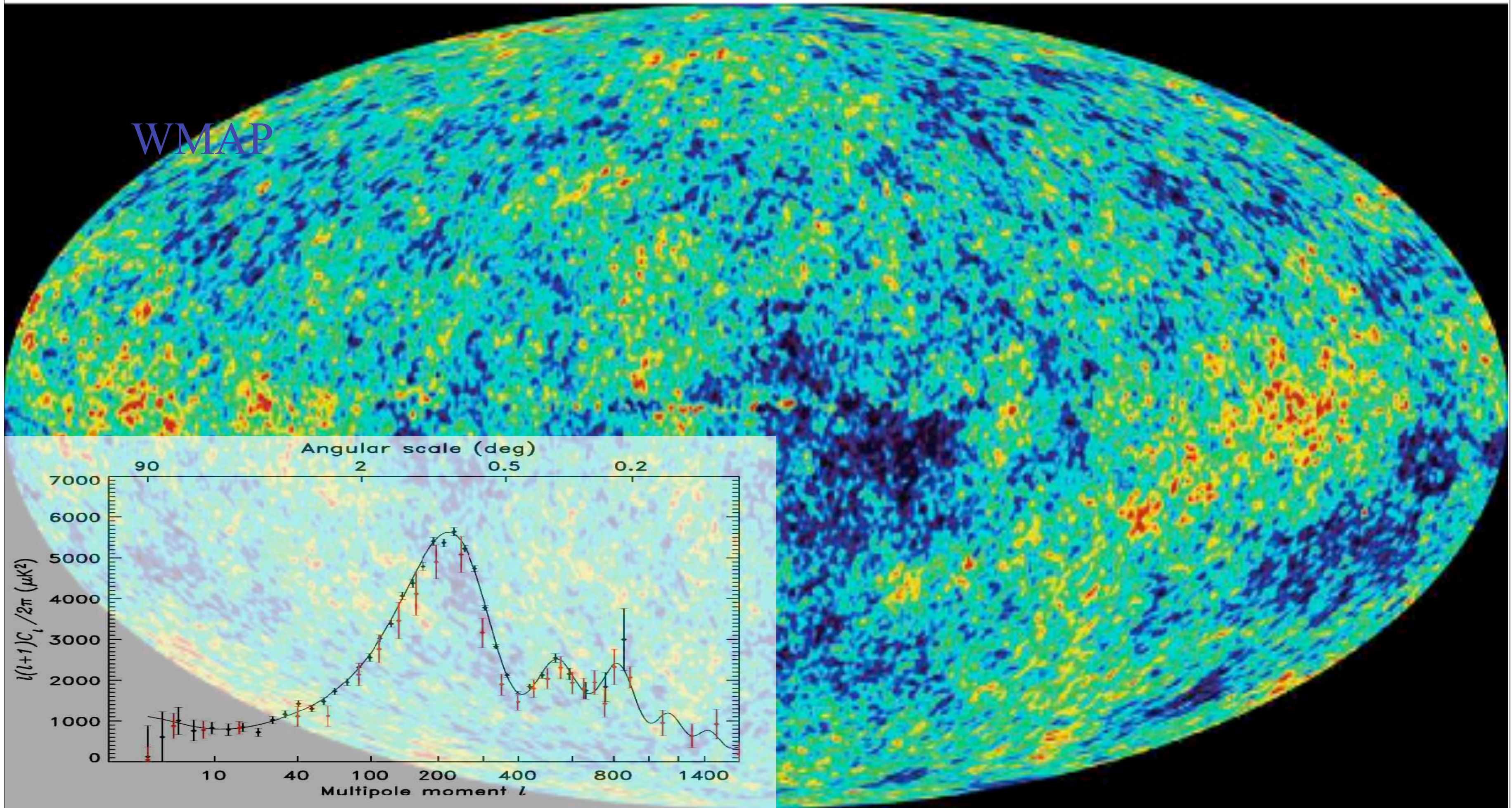
•Nine Temperature maps at 30,44,70,100,143,217,353,545,857 GHz + Polarization

•Angular resolutions: 33', 24', 14', 10', 7.1', 5', 5', 5', 5'

==> DATA Released in 2013

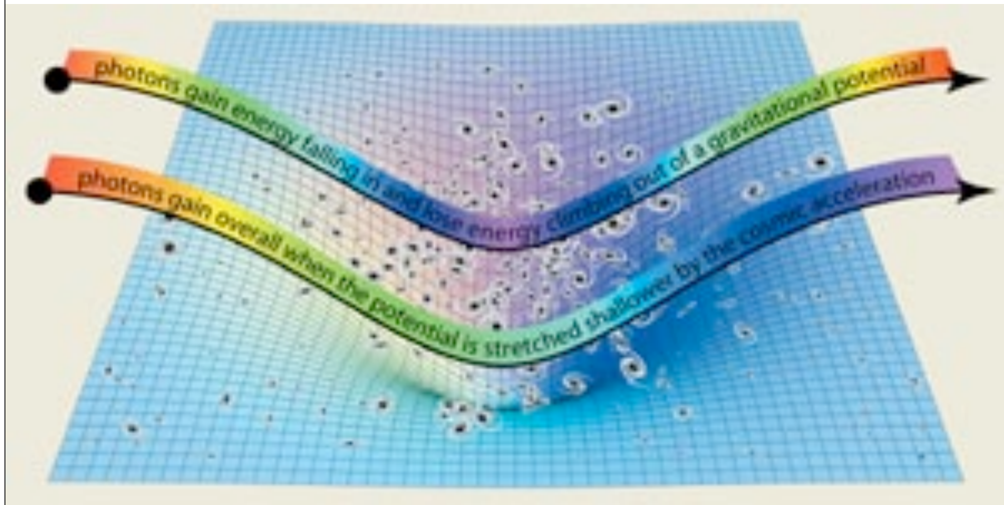

COBE
WMAP
W-band temperature anisotropy
Internal Linear Combination of 5 bands, smoothed
PLANCK
Simulated temperature anisotropy
Simulated temperature and polarization anisotropy

The CMB exhibits Fluctuations



The Cosmic Microwave Background (CMB) is a relic radiation (with a temperature equals to 2.726 Kelvin) emitted 13 billion years ago when the Universe was about 370000 years old.

Integrated Sachs-Wolfe Effect (ISW)



Measure of Time Variation in the Gravitational Potential on **large scales (linear)**

$$\left(\frac{\Delta T}{T}\right)_{ISW} = -2\int \frac{d\Phi}{d\eta}\, d\eta$$

Detect by cross-correlating with local tracers of mass

Detection implies	→	$\Omega_{DE} \neq 0$ (flat prior)
		Open Universe
		Deviations from GR on large scales

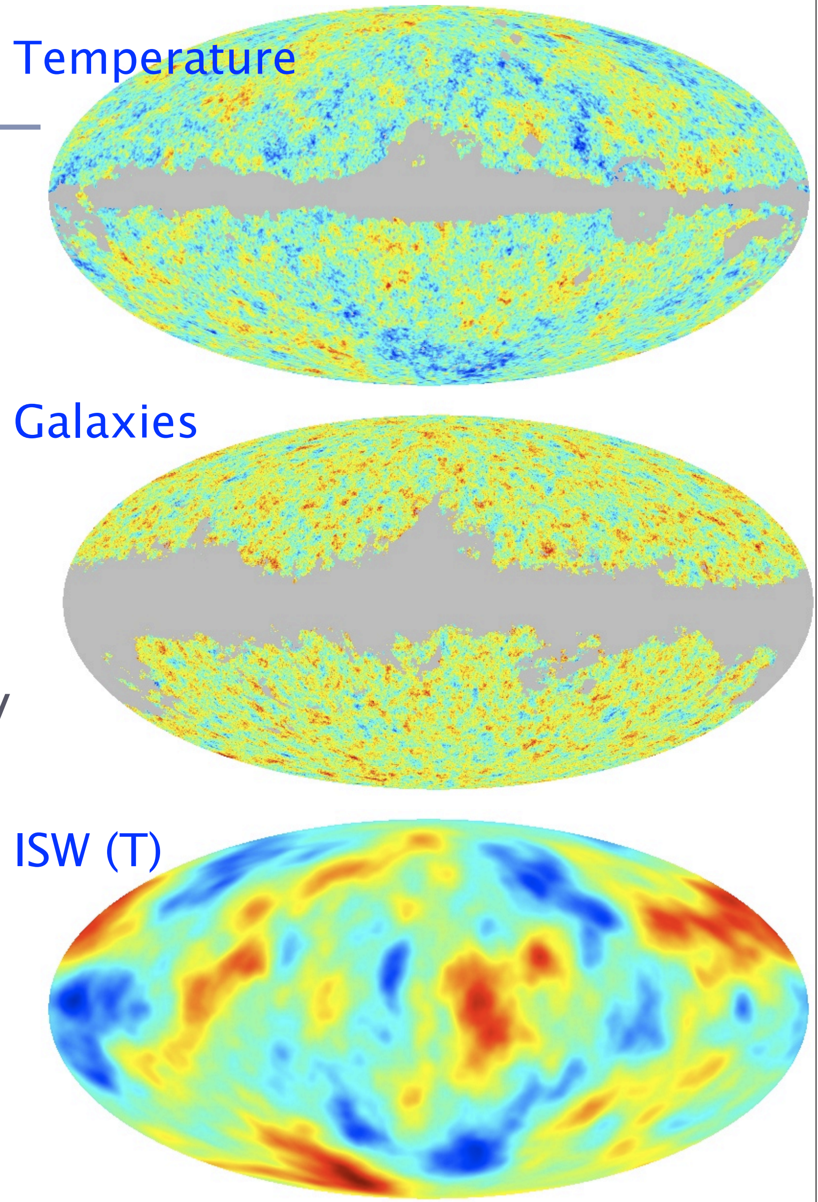
F.-X. Dupe, A. Rassat, J.-L. Starck, M. J. Fadili , "Measuring the Integrated Sachs-Wolfe Effect", arXiv:1010.2192, Astronomy & Astrophysics 534, A51+, 2011.

ISW Reconstruction

- Previously: Cross-Correlate <Tg>

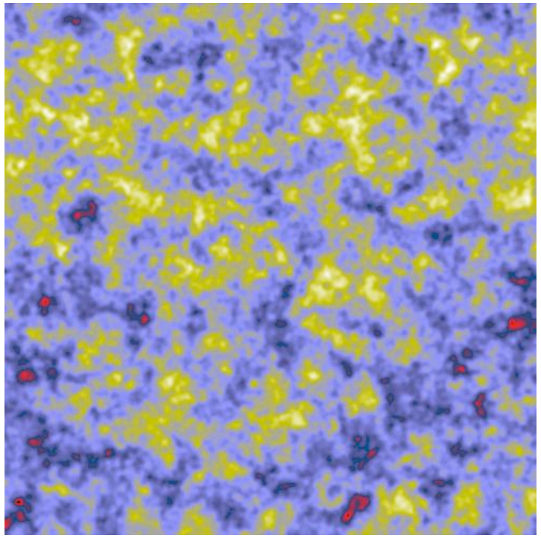
$$T_{\rm CMB}^{\rm obs} = T_{\rm primordial} + \alpha T_{\rm ISW}$$

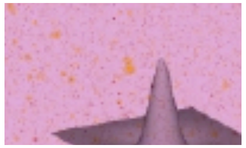
- Reconstruct part of Temperature map due to ISW
  - Reconstruct large scale secondary anisotropies
  - Due to one or several galaxy distributions in foreground
  - Recover primordial T at large scales
- Detection tricky ➔ Reconstruction complex problem



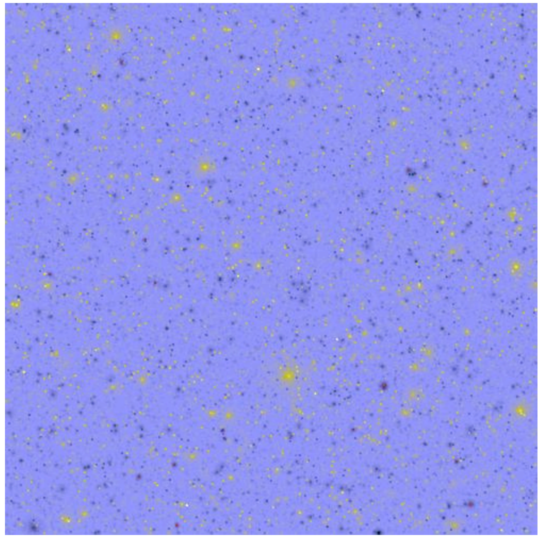
Cosmic Strings

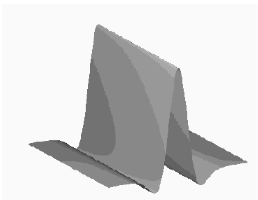
CMB



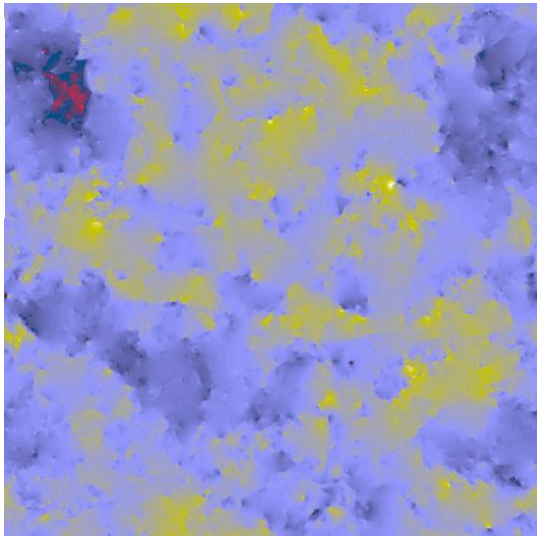


SZ

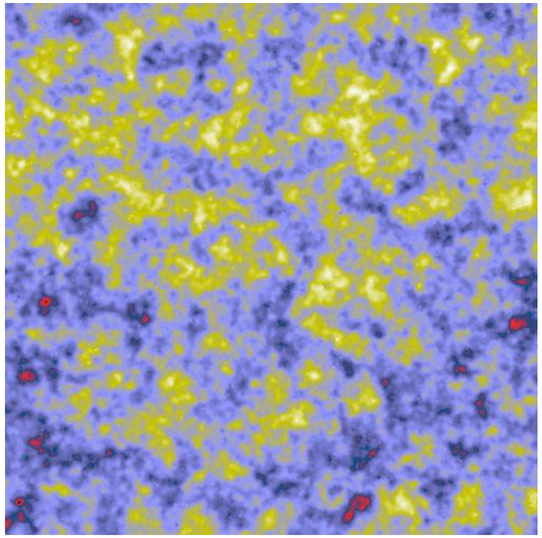




CS



Total

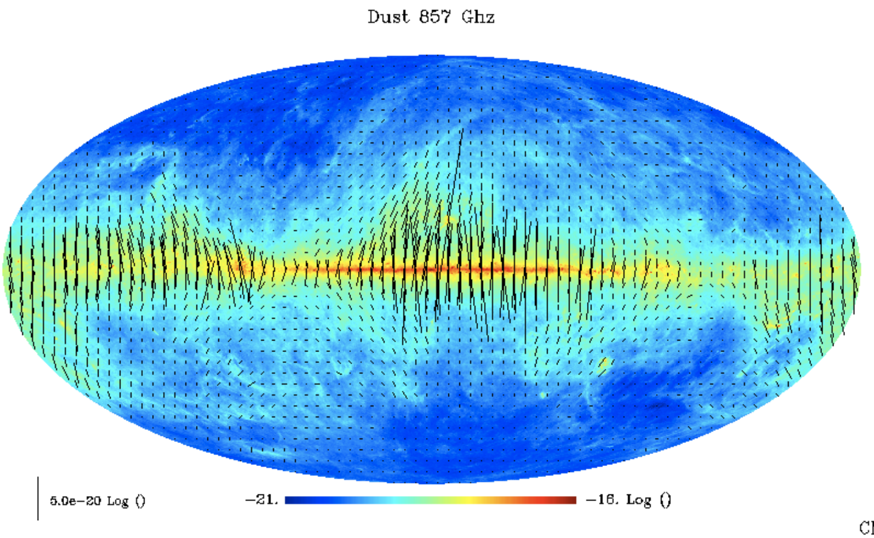


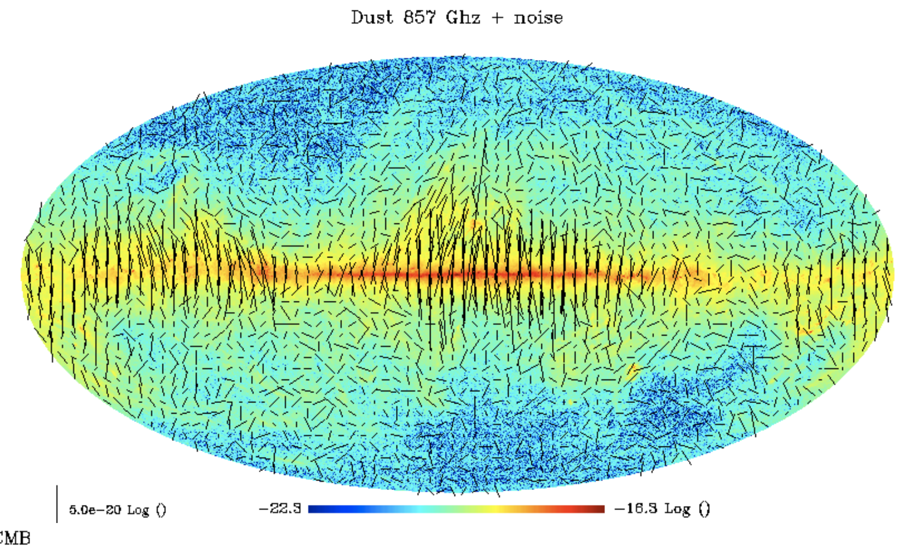
•J.-L. Starck, N. Aghanim and O. Forni, ***"Detecting Cosmological non-Gaussian Signatures by Multi-scale Methods"***, A&A, 416, 9--17, 2004.

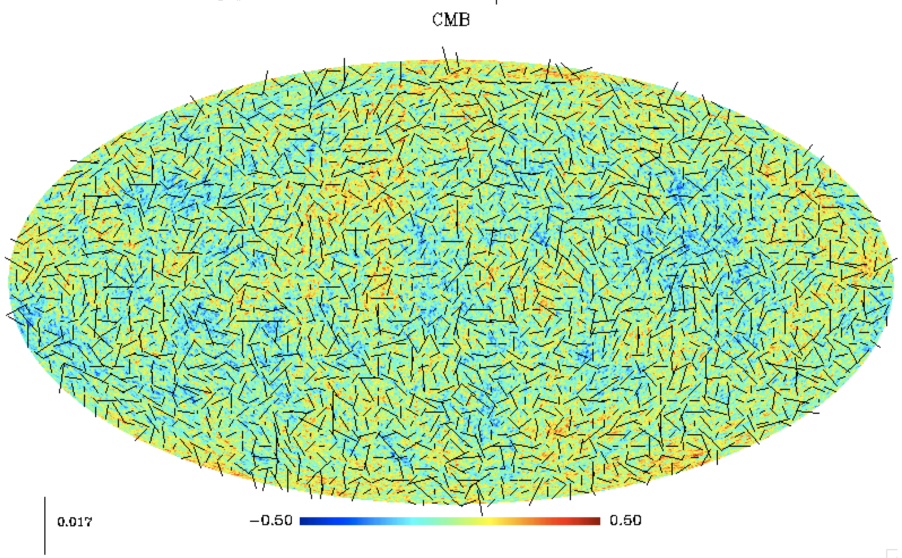
•J. Jin, J.-L. Starck, D.L. Donoho, N. Aghanim and O. Forni, ***"Cosmological Non-Gaussian Signatures Detection: Comparison of Statistical Tests"***, Eurasip Journal on Applied Signal Processing, 15 pp 2470-2485, 2005.

PLANCK SIMULATED POLARIZED DATA:

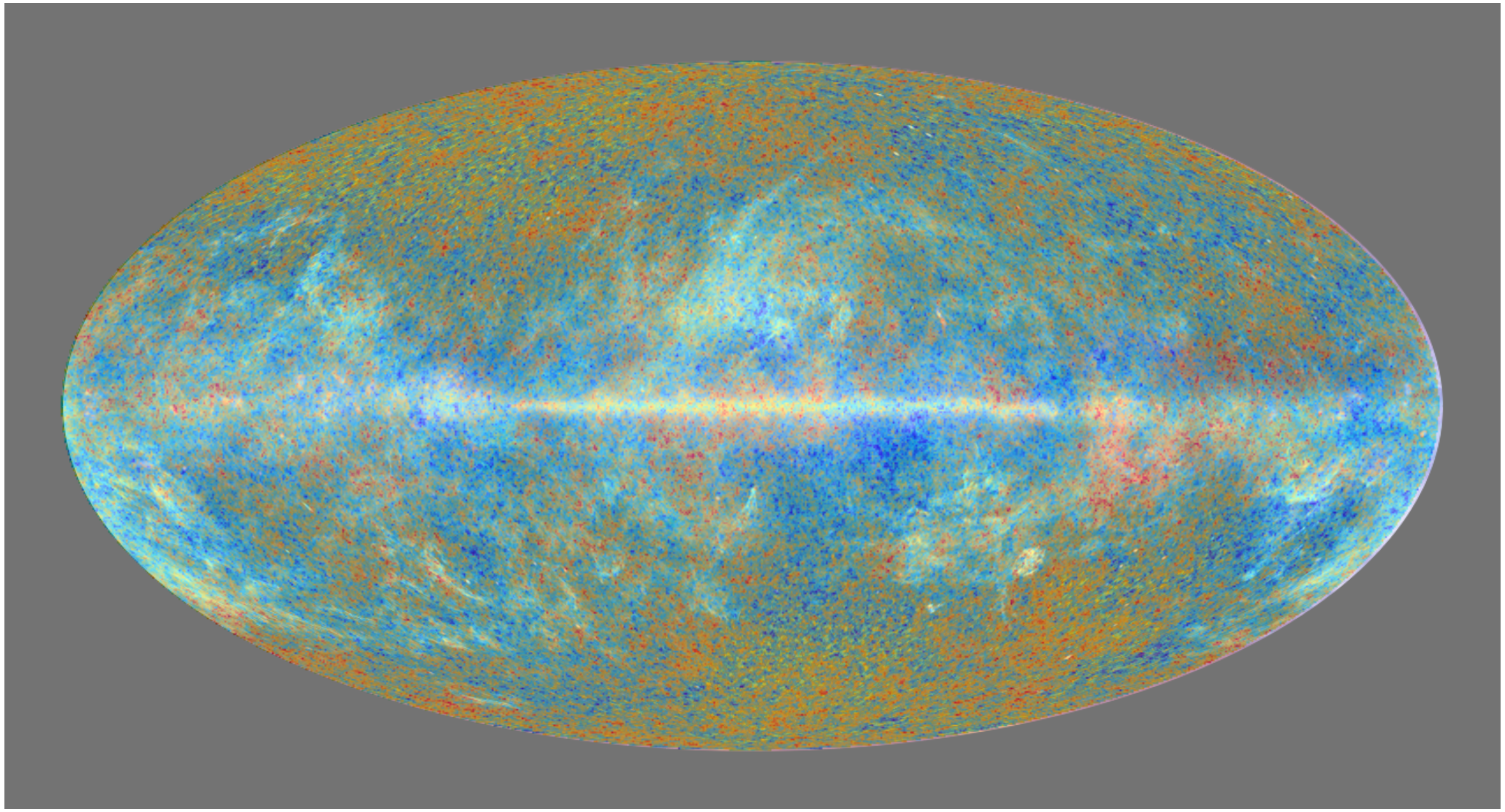
Magnitude $P = \sqrt{Q^2 + V^2}$ Orientation $\alpha = \arctan\left(U/Q\right)$







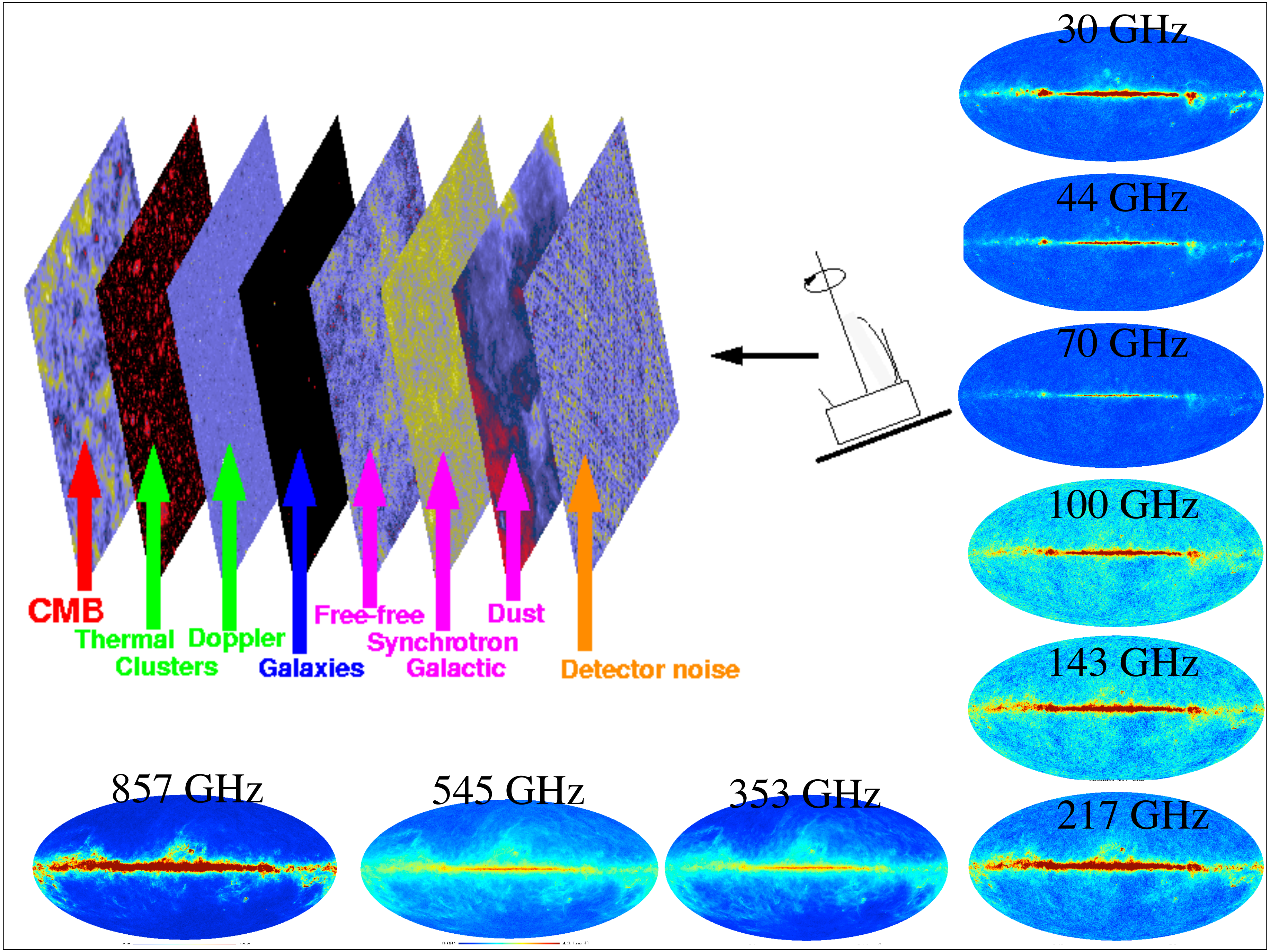
CMB simulated map

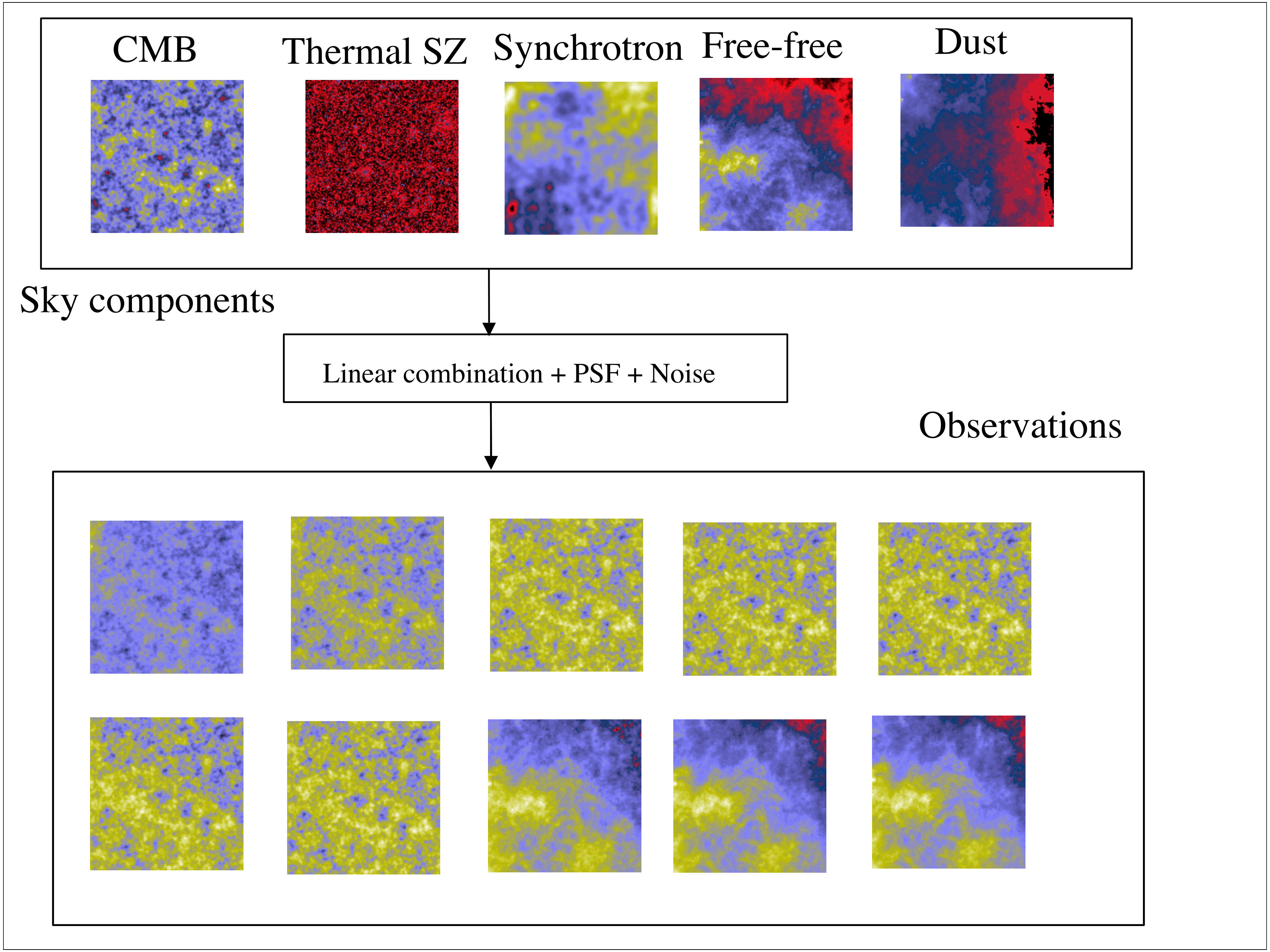


The importance of Source Separation

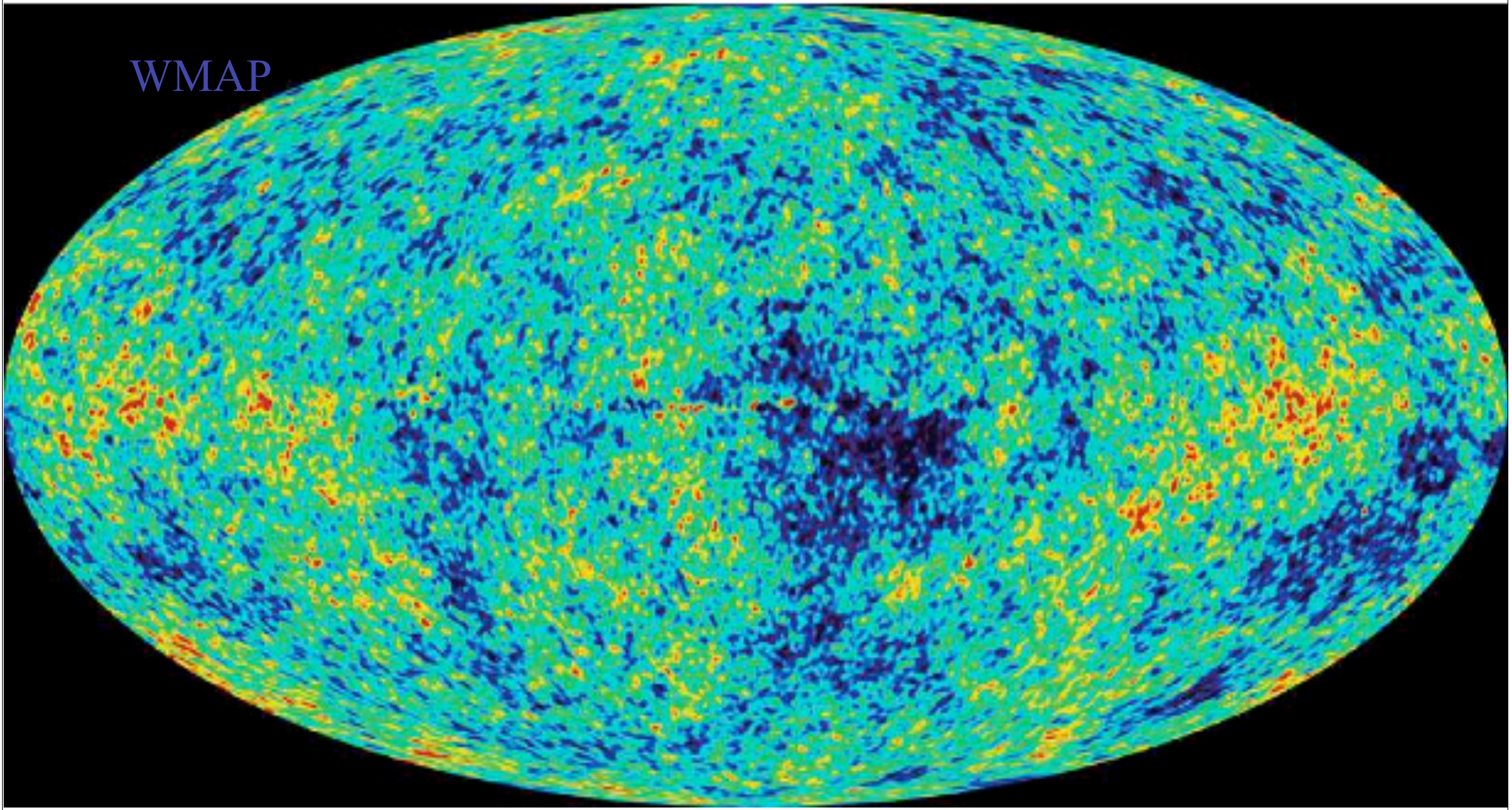
Extra foregrounds are superimposed with the CMB !!!

Point sources, galactic foregrounds, ... etc


30 GHz
44 GHz
70 GHz
100 GHz
143 GHz
217 GHz
857 GHz
545 GHz
353 GHz
CMB
Thermal
Clusters
Doppler
Galaxies
Free-free
Synchrotron
Galactic
Dust
Detector noise


CMB
Thermal SZ
Synchrotron
Free-free
Dust
Sky components
Linear combination + PSF + Noise
Observations

The CMB exhibits Fluctuations

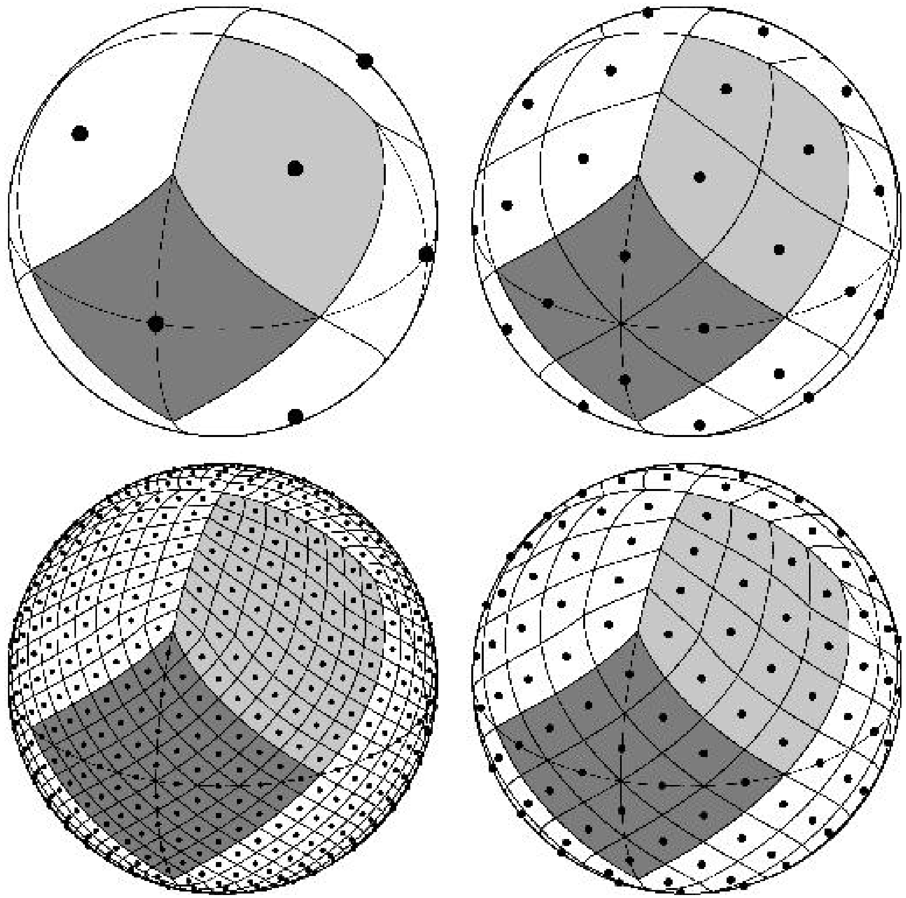


The Cosmic Microwave Background (CMB) is a relic radiation (with a temperature equals to 2.726 Kelvin) emitted 13 billion years ago when the Universe was about 370000 years old.

Healpix

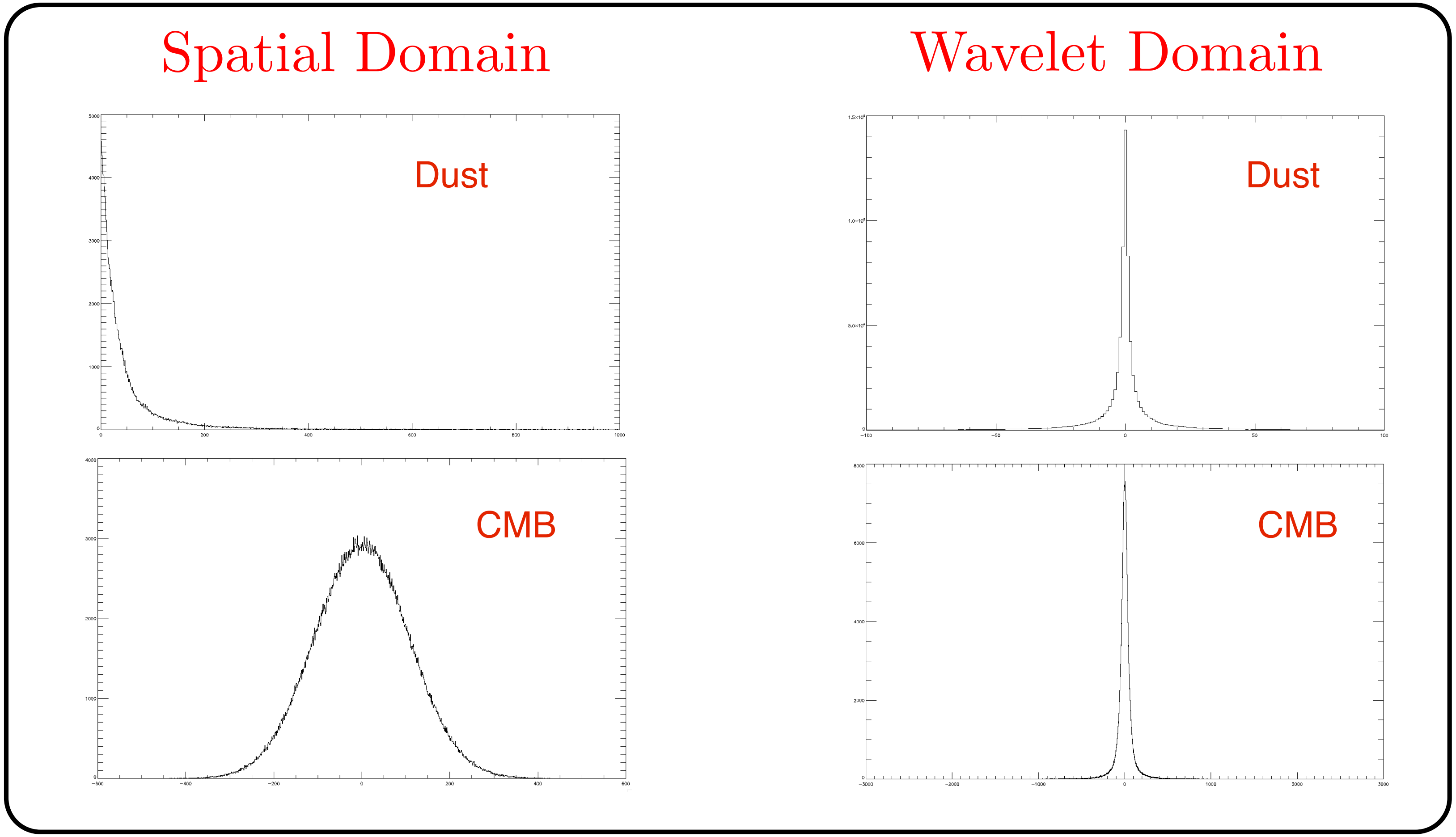
K.M. Gorski et al., 1999, astro-ph/9812350,
http://www.eso.org/science/healpix

- Pixel = Rhombus
- Same Surfaces
- For a given latitude : regularly spaced
- Number of pixels:
  $12 \times (N_{sides})^2$
- Included in the software:
  - Anafast
  - Synfast



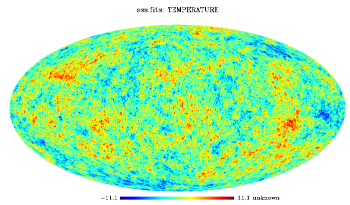
Is Sparse models adapted to PLANCK Data ?

- Components are sparse in a wavelet dictionary



Isotropic Undecimated Wavelet on the Sphere

Wavelets, Ridgelets and Curvelets on the Sphere, Astronomy & Astrophysics, 446, 1191-1204, 2006.



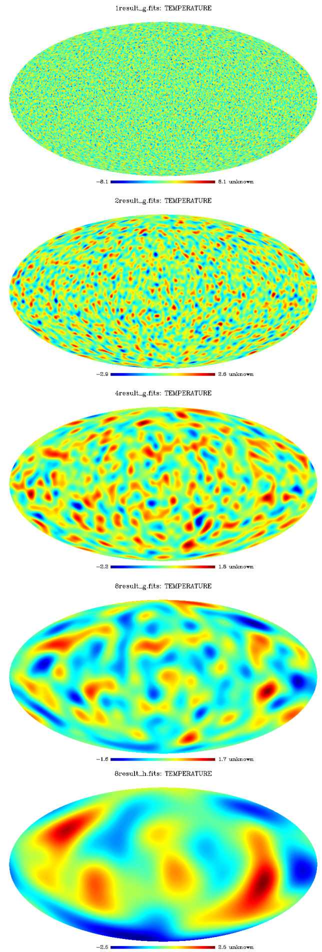
Undecimated
Wavelet Transform

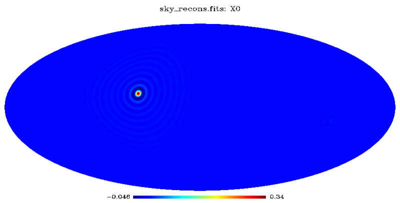
j=1

j=2

j=3

j=4





$$\hat{\psi}_{\frac{l_c}{2^j}}(l,m) = \hat{\phi}_{\frac{l_c}{2^{j-1}}}(l,m) - \hat{\phi}_{\frac{l_c}{2^j}}(l,m)$$

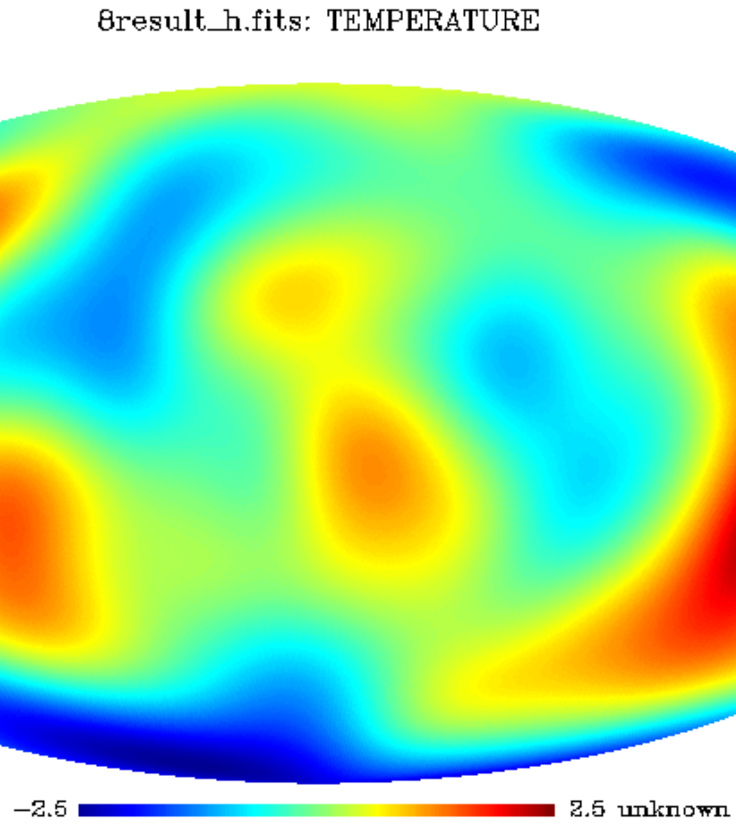
$$\hat{H}_j(l,m) = \begin{cases} \dfrac{\hat{\phi}_{\frac{l_c}{2^{j+1}}}(l,m)}{\hat{\phi}_{\frac{l_c}{2^j}}(l,m)} & \text{if } l < \frac{l_c}{2^{j+1}} \quad \text{and} \quad m = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{G}_j(l,m) = \begin{cases} \dfrac{\hat{\psi}_{\frac{l_c}{2^{j+1}}}(l,m)}{\hat{\phi}_{\frac{l_c}{2^j}}(l,m)} & \text{if } l < \frac{l_c}{2^{j+1}} \quad \text{and} \quad m = 0 \\ 1 & \text{if } l \geq \frac{l_c}{2^{j+1}} \quad \text{and} \quad m = 0 \\ 0 & \text{otherwise} \end{cases}$$

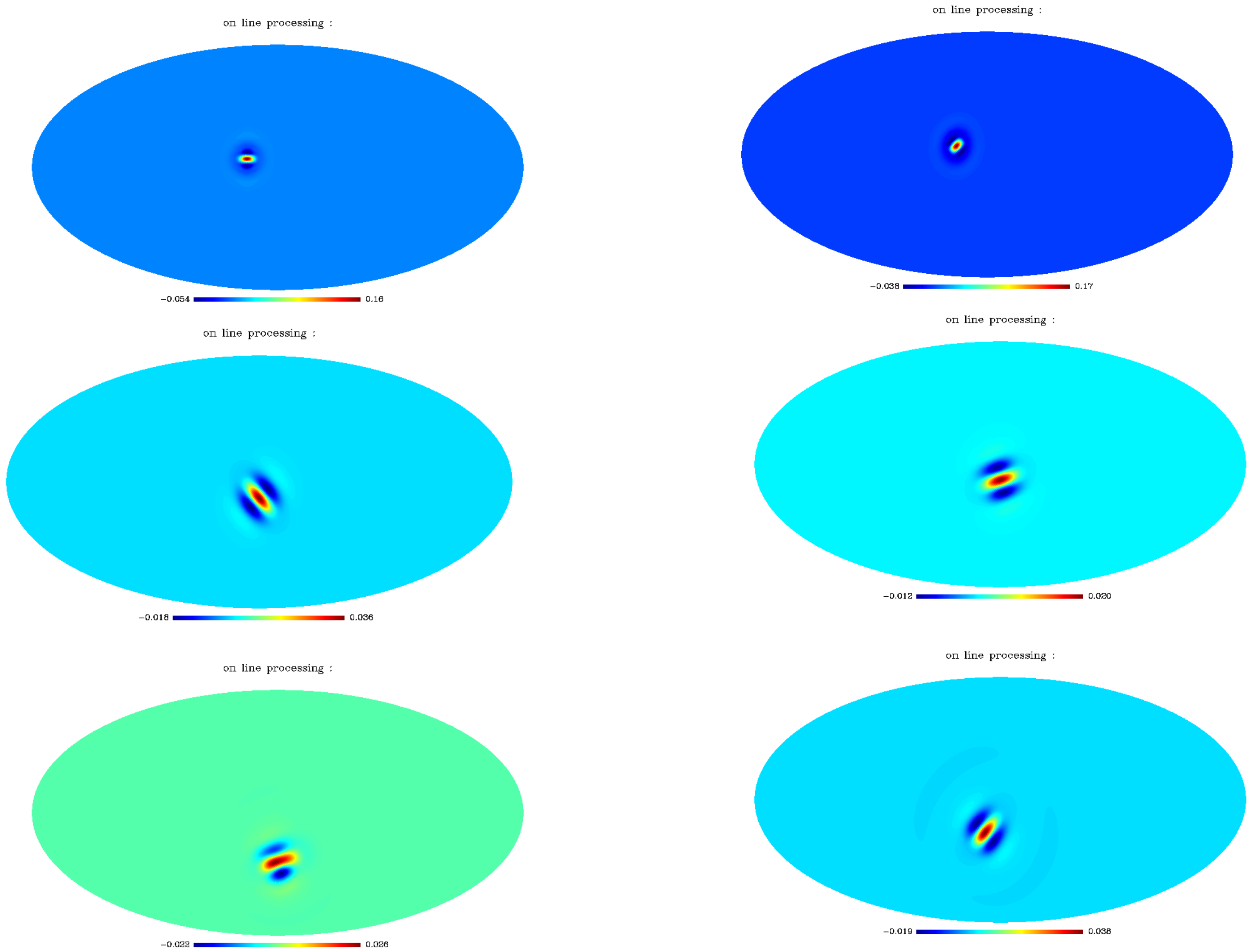
$$\hat{c}_{j+1}(l,m) = \widehat{H}_j(l,m)\hat{c}_j(l,m)$$

$$\hat{w}_{j+1}(l,m) = \widehat{G}_j(l,m)\hat{c}_j(l,m)$$

$$c_0(\vartheta,\varphi) = c_J(\vartheta,\varphi) + \sum_{j=1}^{J} w_j(\vartheta,\varphi)$$

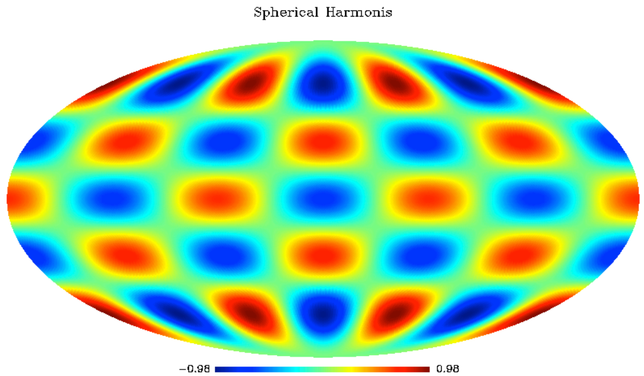

8result_h.fits: TEMPERATURE
−2.5
2.5 unknown

Curvelet functions on the sphere

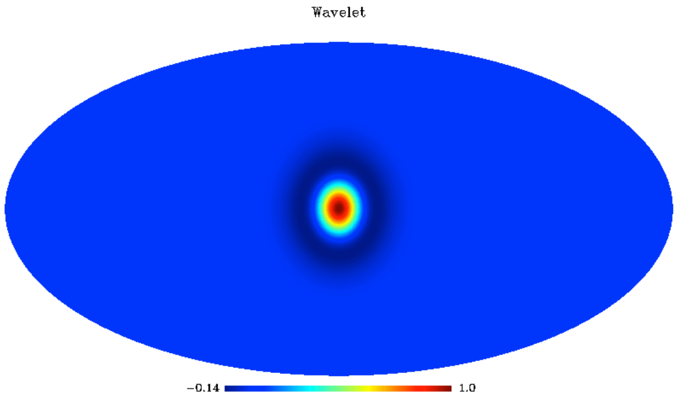


Dictionaries

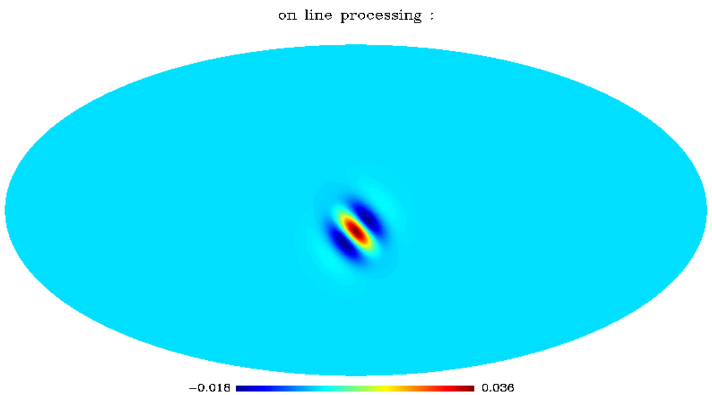
Spherical Harmonics



Wavelets

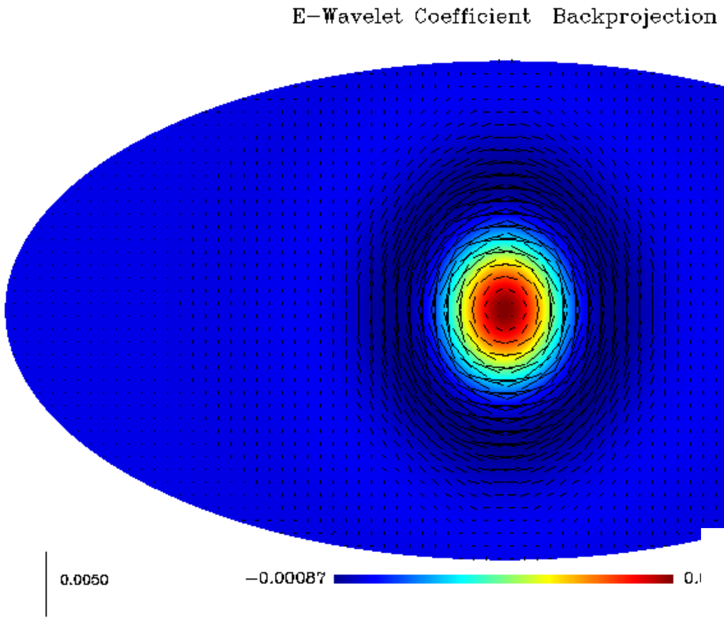


Curvelets

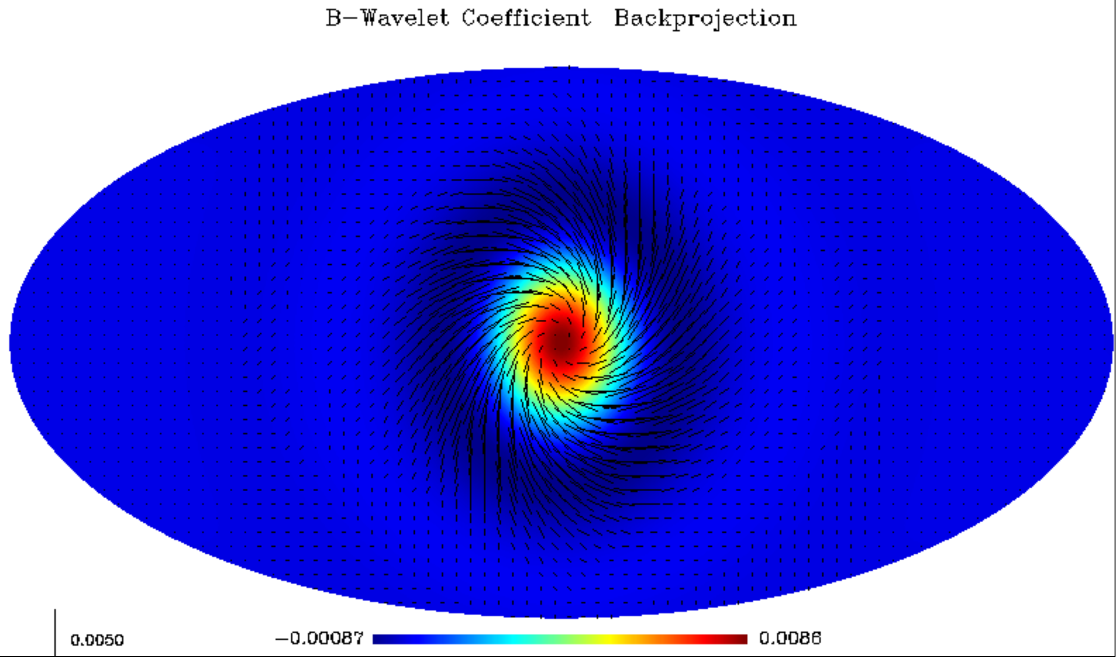


Polarized Dictionary: E/B Polarized Wavelet

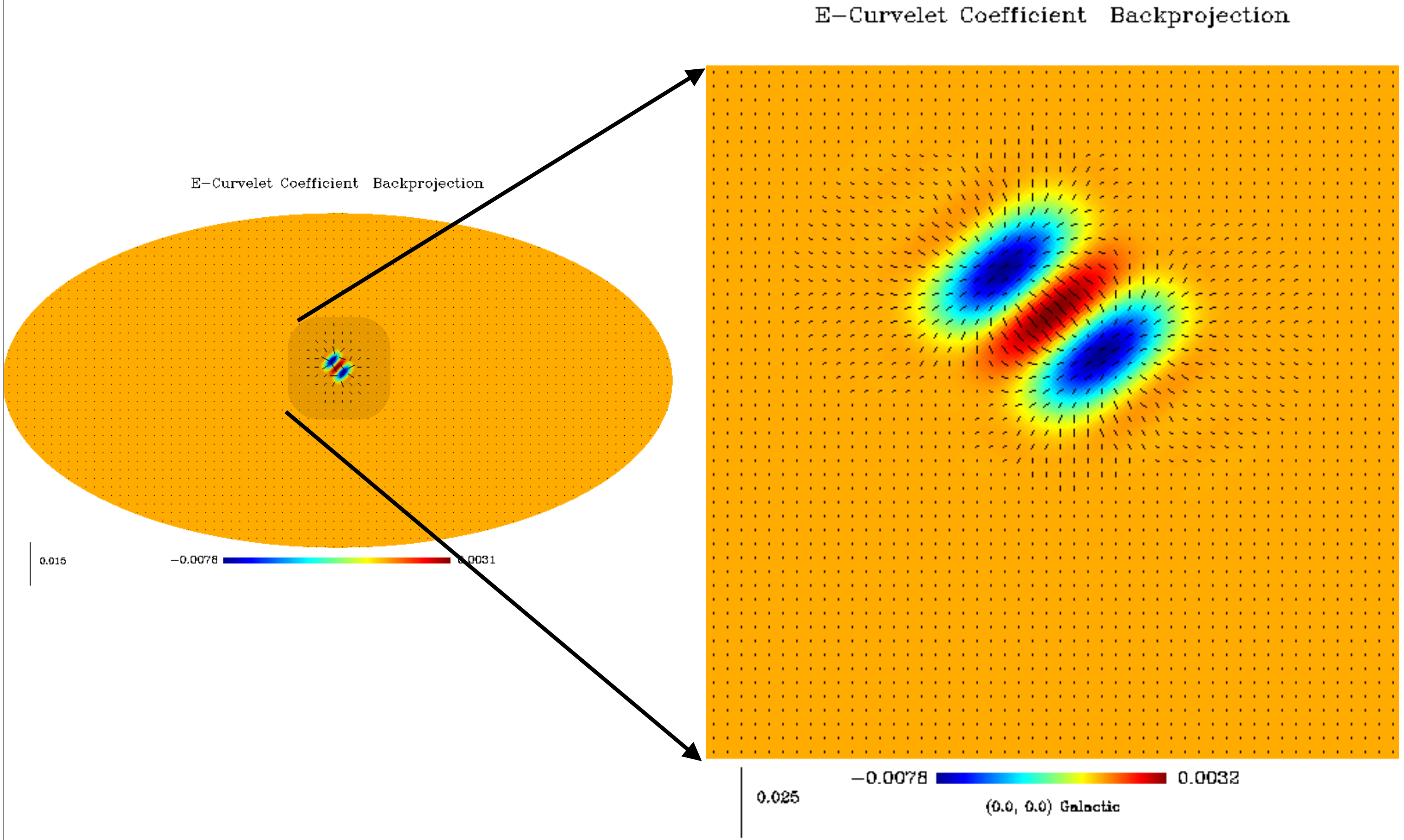
J.-L. Starck, Y. Moudden and J. Bobin, "Polarized Wavelets and Curvelets on the Sphere", Astronomy and Astrophysics, 497, 3, pp 931--943, 2009.



j=4



Curvelet and E/B Mode Decomposition



Polarized Data Denoising

$$Q(\theta,\phi) = \sum_{l,m} c^E_{J,l,m} Z^+_{l,m} + i c^B_{J,l,m} Z^-_{l,m} + \sum_j \sum_{l,m} \tilde{w}^E_{j,l,m} Z^+_{l,m} + i \tilde{w}^B_{j,l,m} Z^-_{l,m}$$

$$U(\theta,\phi) = \sum_{l,m} c^B_{J,l,m} Z^+_{l,m} - i c^E_{J,l,m} Z^-_{l,m} + \sum_j \sum_{l,m} \tilde{w}^B_{j,l,m} Z^+_{l,m} - i \tilde{w}^E_{j,l,m} Z^-_{l,m}$$

Where

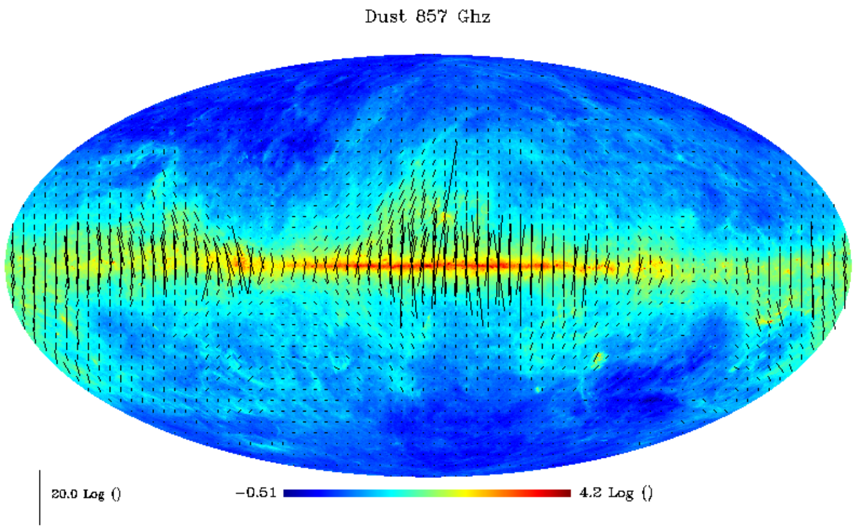
$$\tilde{w}^E_{j,k} = \delta(w^E_{j,k})$$

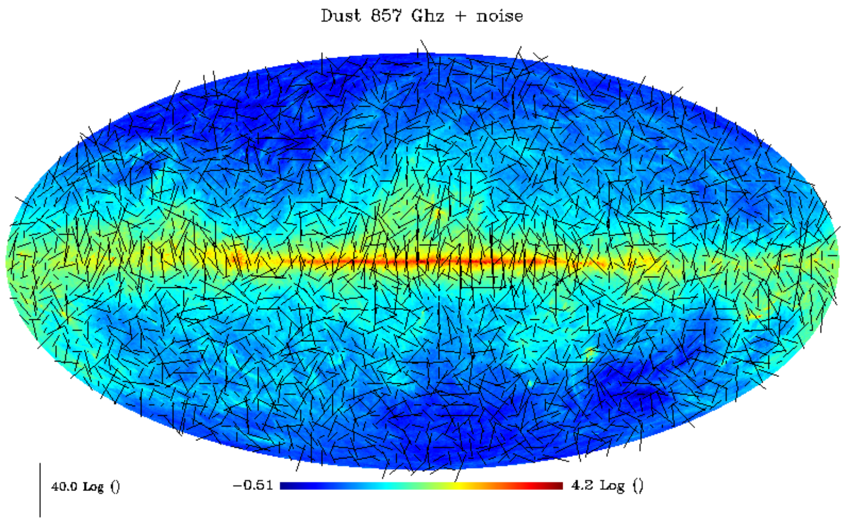
$$\tilde{w}^B_{j,k} = \delta(w^B_{j,k})$$

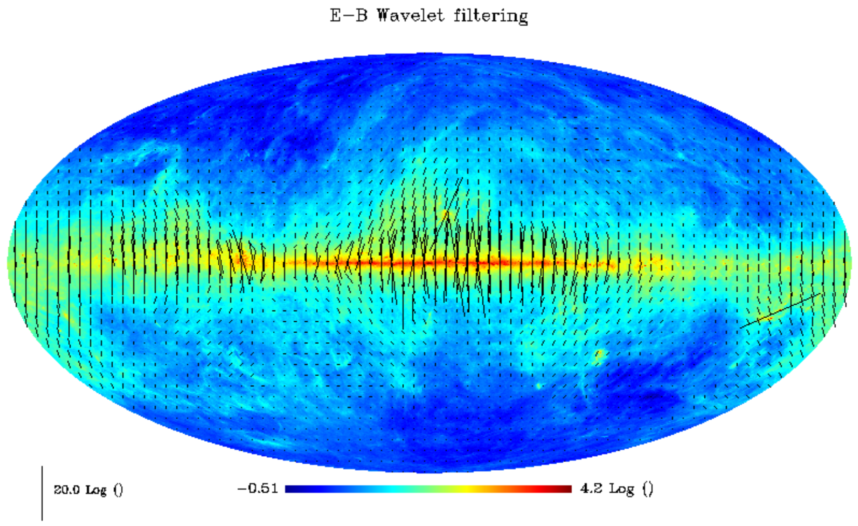
Hard thresholding corresponds to the following non linear operation:

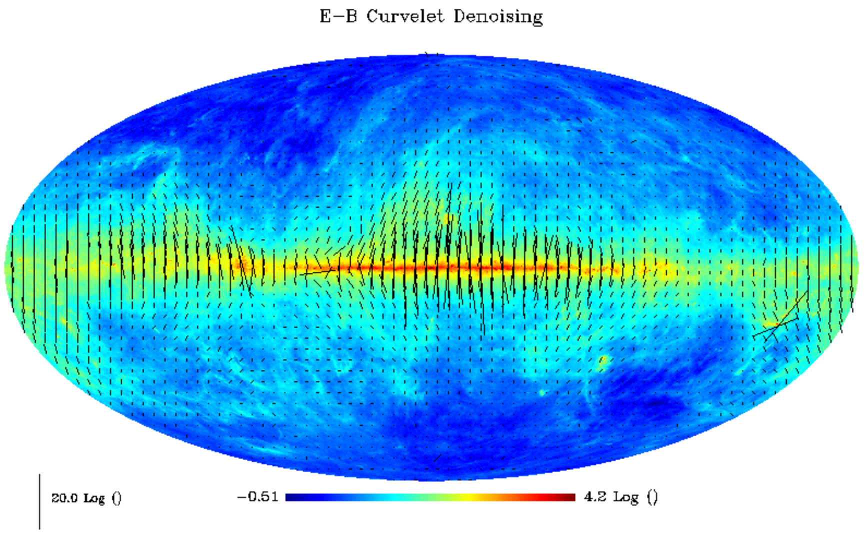
$$\tilde{w}_{j,k} = \begin{cases} w_{j,k} & \text{if } \mid w_{j,k} \mid \geq T_j \\ 0 & \text{otherwise} \end{cases}$$

Polarized Data Denoising









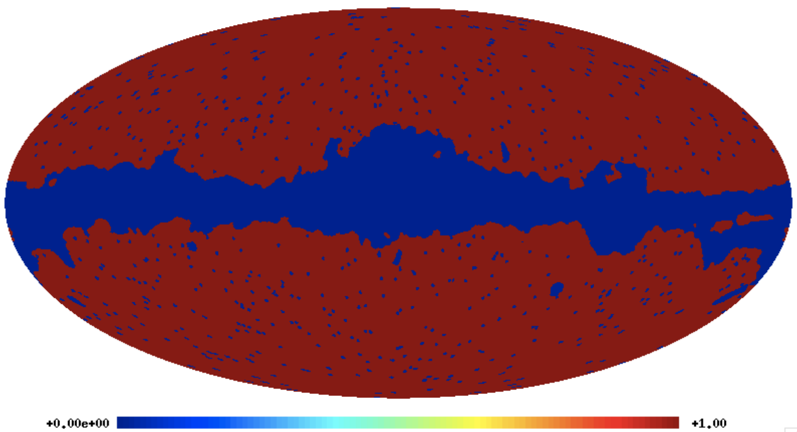
Interpolation of Missing Data: Sparse Inpainting

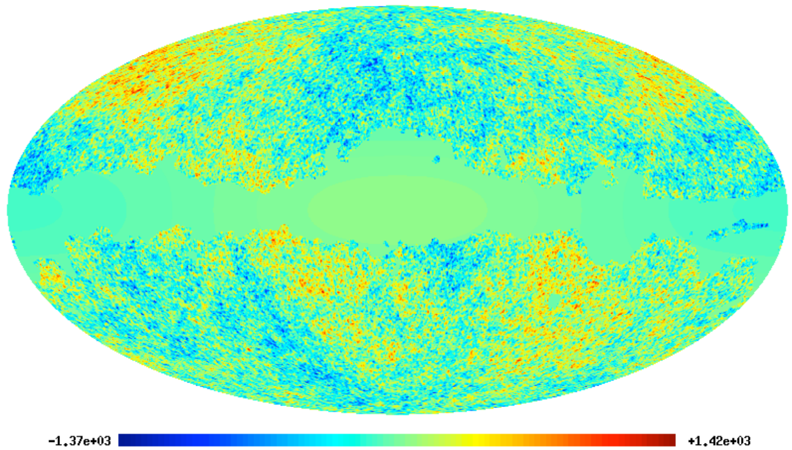
•*M. Elad, J.-L. Starck, D.L. Donoho, P. Querre, "Simultaneous Cartoon and Texture Image Inpainting using Morphological Component Analysis (MCA)", ACHA, Vol. 19, pp. 340-358, 2005.*

• P. Abrial, Y. Moudden, J.L. Starck, M.J. Fadili, J. Delabrouille, and M. Nguyen, "CMB Data Analysis and Sparsity" , **Statistical Methodology** , Vol 5, No 4, pp 289-298, 2008.

$$\min_{\alpha} \|\alpha\|_p \quad \text{subject to} \quad Y = M\Phi\alpha$$

Where M is the mask: M(i,j) = 0 ==> missing data
M(i,j) = 1 ==> good data





Sparse-Inpainting preserves the ISW and the Weak Lensing signal.

L. Perotto, J. Bobin, S. Plaszczynski, J.-L. Starck, and A. Lavabre, "Reconstruction of the CMB lensing for Planck", 5109 , A4, A&A, 2010.
F.-X. Dupe, A. Rassat, J.-L. Starck, M. J. Fadili , "Measuring the Integrated Sachs-Wolfe Effect", A&A, arXiv:1010.2192, 534, A51+, 2011.

Theoretical justification through the sampling theory of Compressed Sensing ?

Rauhut and Ward, "Sparse Legendre expansion via l1 minimization", Constructive Approximation journal, 2010.

Semi-Blind Source Separation

Standard approaches amount to model each observation as a linear mixture of elementary components (i.e. CMB, SZ, Synchrotron, Free-Free, Dust ...) :

$$\forall i;\ x_i = \sum_j a_{ij} s_j + n_j$$

Which can be recast as: $$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N}$$

Blind source separation:
The objective is to estimate both A and S simultaneously !!

- CMB, SZ and Free-Free emission : their electromagnetic spectrum is well known *(i.e. the related columns of A are known and fixed).*

- Synchrotron emission : rank-1 assumption / its electromagnetic spectrum is a power law with an unknown spectral index.

We have nine channels and we search for nine sources:

4 sources are modeled and 5 are not modeled.

Morpho-Spectral Diversity

Data: $X = [x_1, \ldots, x_m]$

Source: $S = [s_1, \ldots, s_n]$

$$X = [x_1, \ldots, x_m] = AS$$

$$x_l = \sum_{i=1}^{n} a_{i,l} s_i$$

$$\min_\alpha \|\alpha\|_p \text{ s.t } \mathbf{X} = \sum_{\gamma \in \Gamma} \alpha_\gamma \psi_\gamma$$

$\mathbf{\Phi_A} = [\mathbf{\Phi_{A,1}}, \mathbf{\Phi_{A,2}}]$ Spatial Dictionary with

$\mathbf{\Phi_S}$ Spectral Dictionary

$$\mathbf{\Psi} = [\mathbf{\Phi_{A,1}} \otimes \mathbf{\Phi_S}, \mathbf{\Phi_{A,2}} \otimes \mathbf{\Phi_S}]$$

Sparse Component Separation: the GMCA Method

- J. Bobin, J.-L. Starck, M.J. Fadili, and Y. Moudden, "Sparsity, Morphological Diversity and Blind Source Separation", IEEE Trans. on Image Processing, Vol 16, No 11, pp 2662 - 2674, 2007.
- .J. Bobin, J.-L. Starck, M.J. Fadili, and Y. Moudden, "Blind Source Separation: The Sparsity Revolution", Advances in Imaging and Electron Physics , Vol 152, pp 221 -- 306, 2008.

Source: $S = [s_1, s_n]$ Data: $X = [x_1, ..., x_n] = AS + N$

We define a dictionary ϕ

==> GMCA searches a sparse solution S in the dictionary ϕ subject to the constraint that the norm $\| X - AS \|^2$ is minimal.

A and S are estimated alternately and iteratively in two steps :

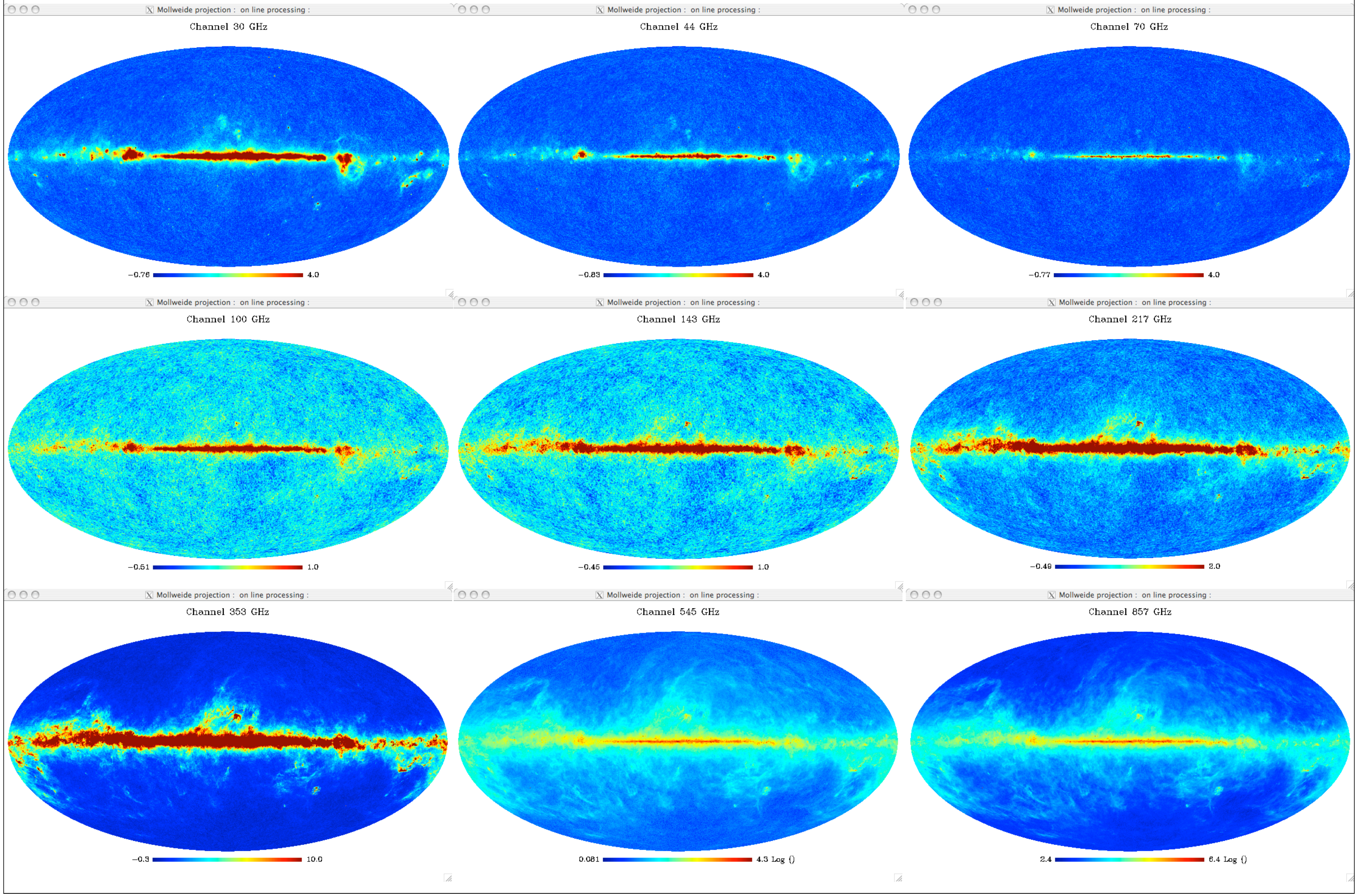
1) Estimate S assuming A is fixed (iterative thresholding) :

$$\{S\} = \text{Argmin}_S \sum_j \lambda_j \|s_j \mathbf{W}\|_1 + \|\mathbf{X} - \mathbf{AS}\|^2_{F,\mathbf{\Sigma}}$$

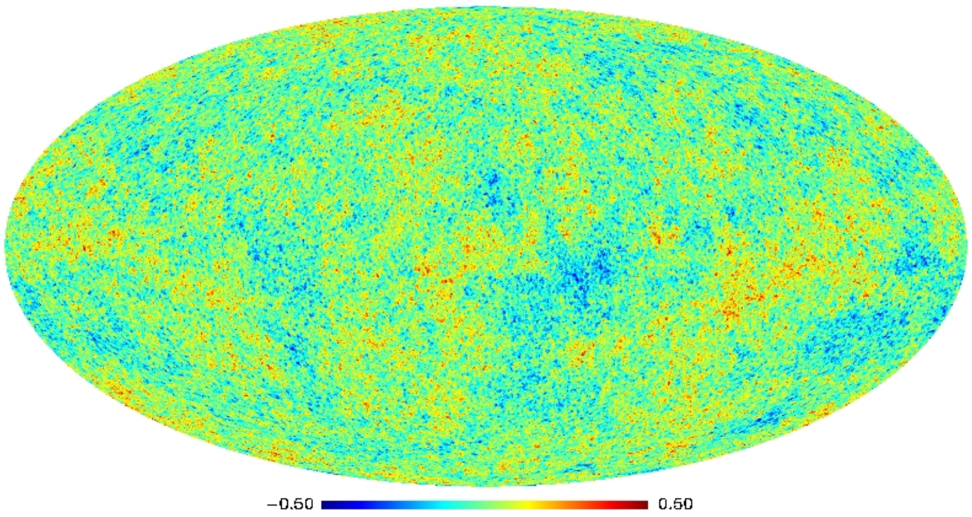
2) Estimate A assuming S is fixed (a simple least square problem) :

$$\{A\} = \text{Argmin}_A \|\mathbf{X} - \mathbf{AS}\|^2_{F,\mathbf{\Sigma}}$$

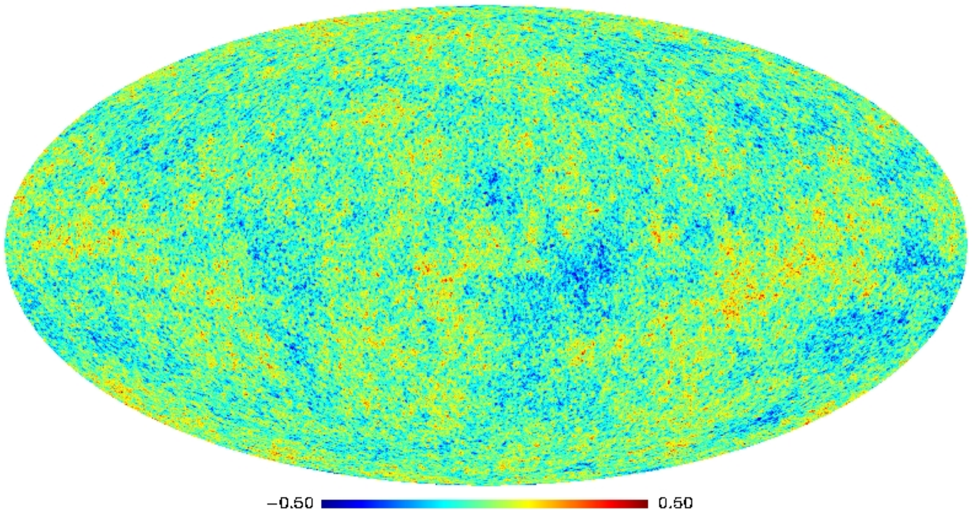
PLANCK Simulated Data



Planck - WG2 - Challenge 2



Input simulated CMB map (mK)



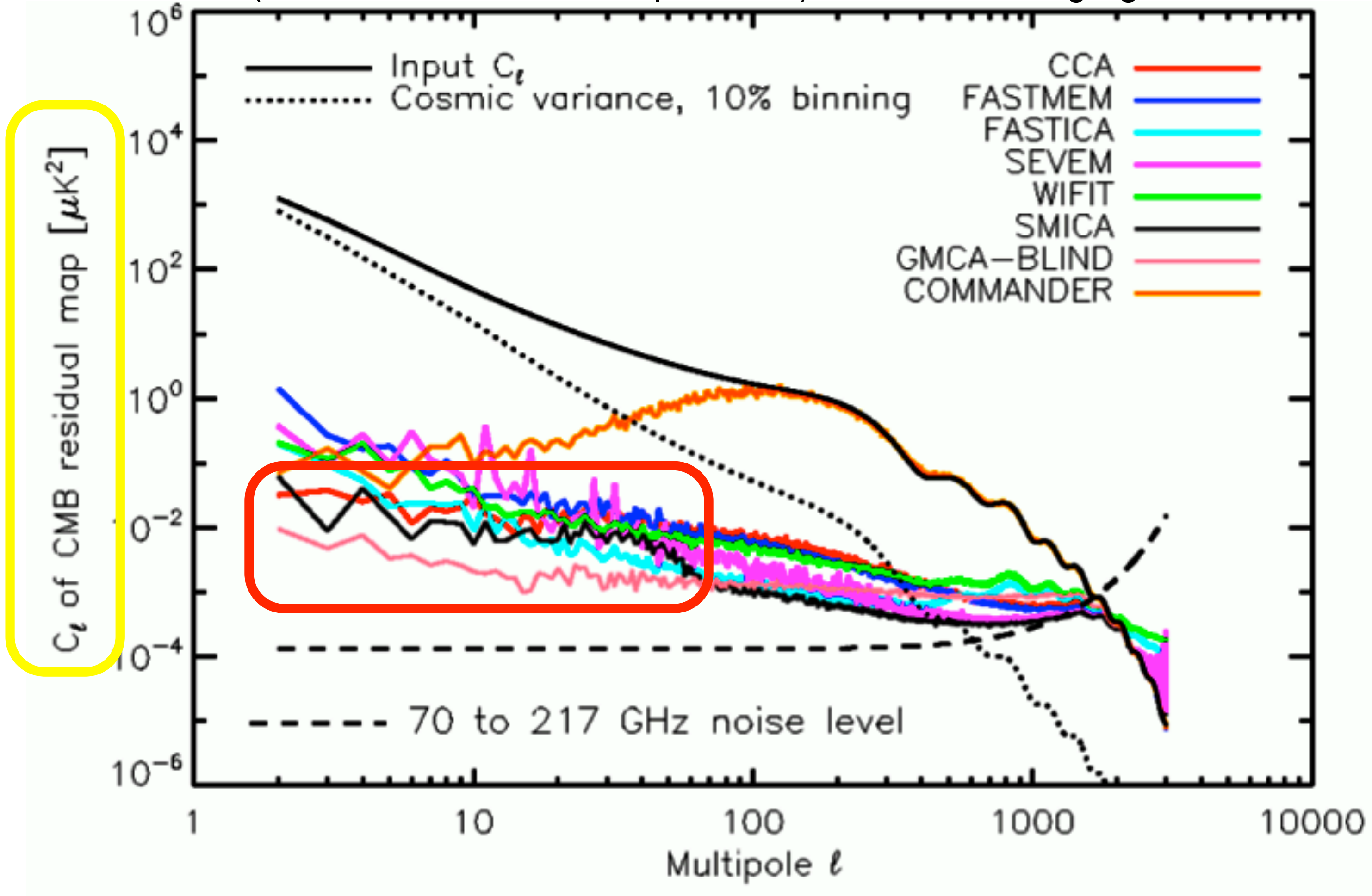
Estimated map with GMCA

Planck - WG2 - Challenger 2

comparaisons - power spectrum

GMCA has the lowest spectral residuals at low l.

Plot shows C_l of (reconstructed CMB – input CMB) evaluated at high galactic latitudes.



- **Sam Leach** (SISSA), June 19, 2008, WG2 meeting, Munich

Limitations

GMCA Model: $Y = A X + N$

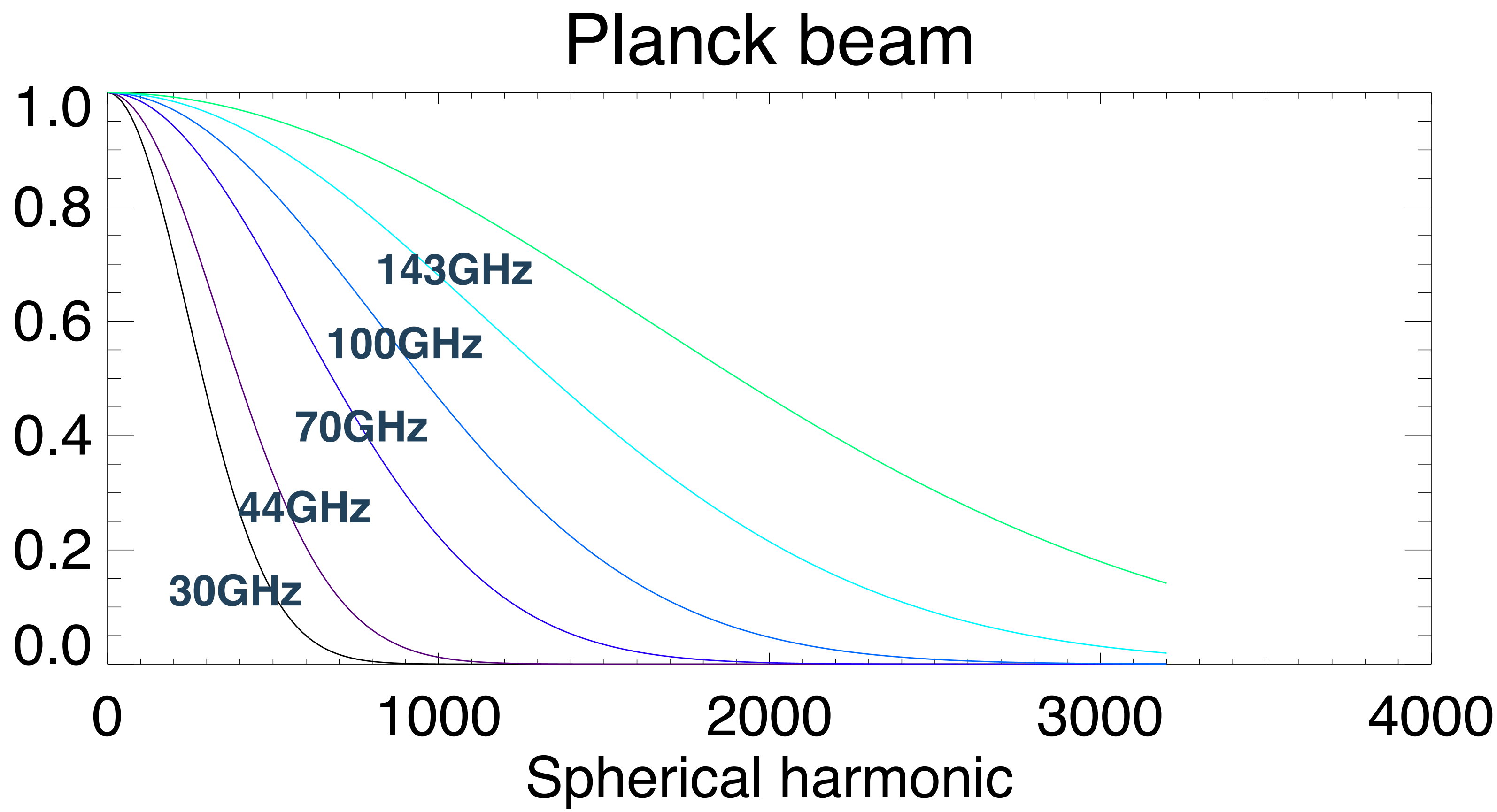
But three main problems:
i) A is spatially variant.
ii) This model does not take into account the beam.
iii) Noise is not homogeneous.

- Limitation of GMCA:

- One matrix to describe the whole sky (i.e. the simplest model !)
- PSF were not taken into account properly
- Non stationary noise.

Component Separation

Planck provides data that do not share the same resolution :



Component Separation: more problems

More formally:

$$\forall i;\ x_i = b_i \star \left(\sum_j a_{ij} s_j \right) + n_i$$

Globally: $\mathbf{X} = \mathcal{H}\left(\mathbf{AS}\right) + \mathbf{N}$

where $\mathcal{H}$ is the multichannel convolution operator

The mixture model no more holds ! $\mathcal{H}$ **is singular !**

Spectral behavior varies spatially for some components (dust, synchroton):

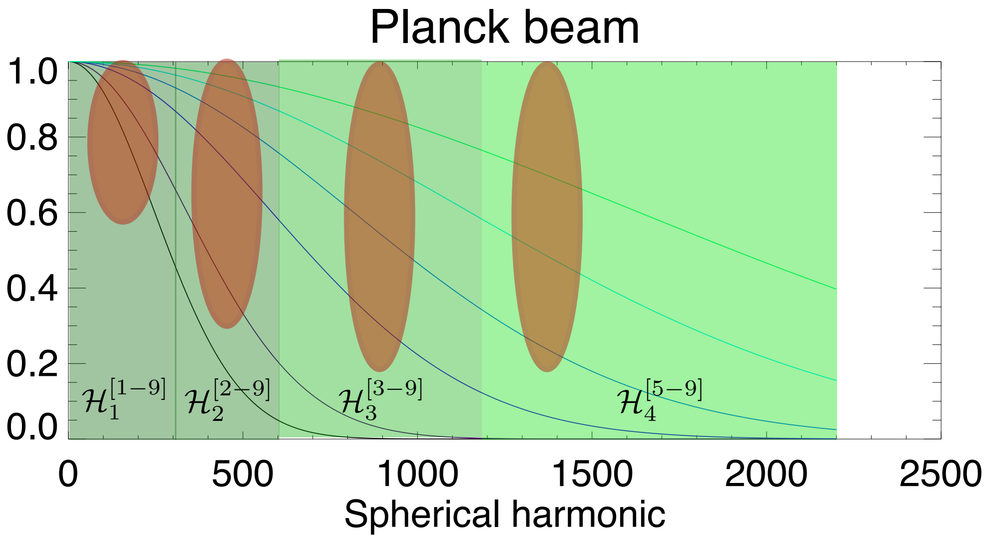
The BEAM problem

1- Work in the spherical harmonic domain (SMICA)
2- Perform the component separation several times:
   . one with all channels up to l=300,
   . Repeat with less channels up to 500, 800, 1200, 3000.
   . Merge all results

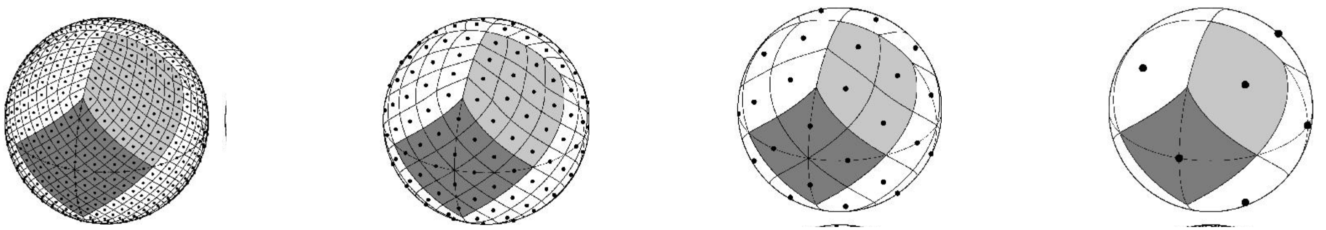
The second approach could be done in much more elegant way using the Wavelet-Vaguelette Decomposition (Donoho, 1995, Abramovich, 1998).

Component Separation

=> Use Wavelets to work at different resolutions:



=> Assume the mixing matrix varies smoothly
Partitionning of the Wavelet Scales



Wavelet-Vaguelette GMCA Decomposition

Inverse problem

$$y = Kf + n \xrightarrow{\text{WVD}} f = \sum_j \sum_k \langle Kf, \Psi_{j,k} \rangle \, \psi_j, k \qquad \text{with } K^* \Psi_{j,k} = \psi_{j,k}$$

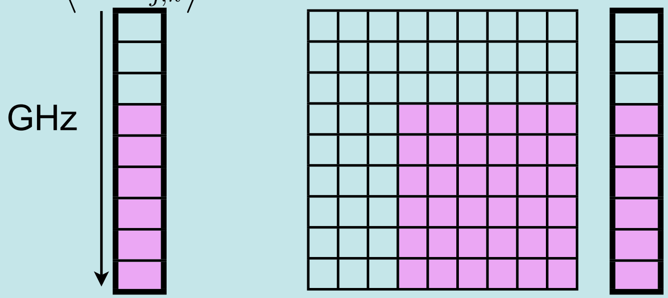
$$\tilde{f} = \sum_j \sum_k \Delta(\langle y, \Psi_{j,k} \rangle) \psi_j, k$$

Multi-channel WVD

$$Y_i = \sum\nolimits_j K_i (A_j X)_i + N_i \longrightarrow \tilde{X}_s = \left[\sum_j \sum_k \tilde{A}_j^+ \left\langle Y_i, \Psi_{j,k}^{(i)} \right\rangle \psi_{j,k} \right]_s$$

$$\text{with } K_i^* \Psi_{j,k}^{(i)} = \psi_{j,k}$$

$\beta_{j,k} = \left\langle Y_i, \Psi_{j,k}^{(i)} \right\rangle$ = A_j $\alpha_{j,k} = \langle X_s, \psi_{j,k} \rangle$

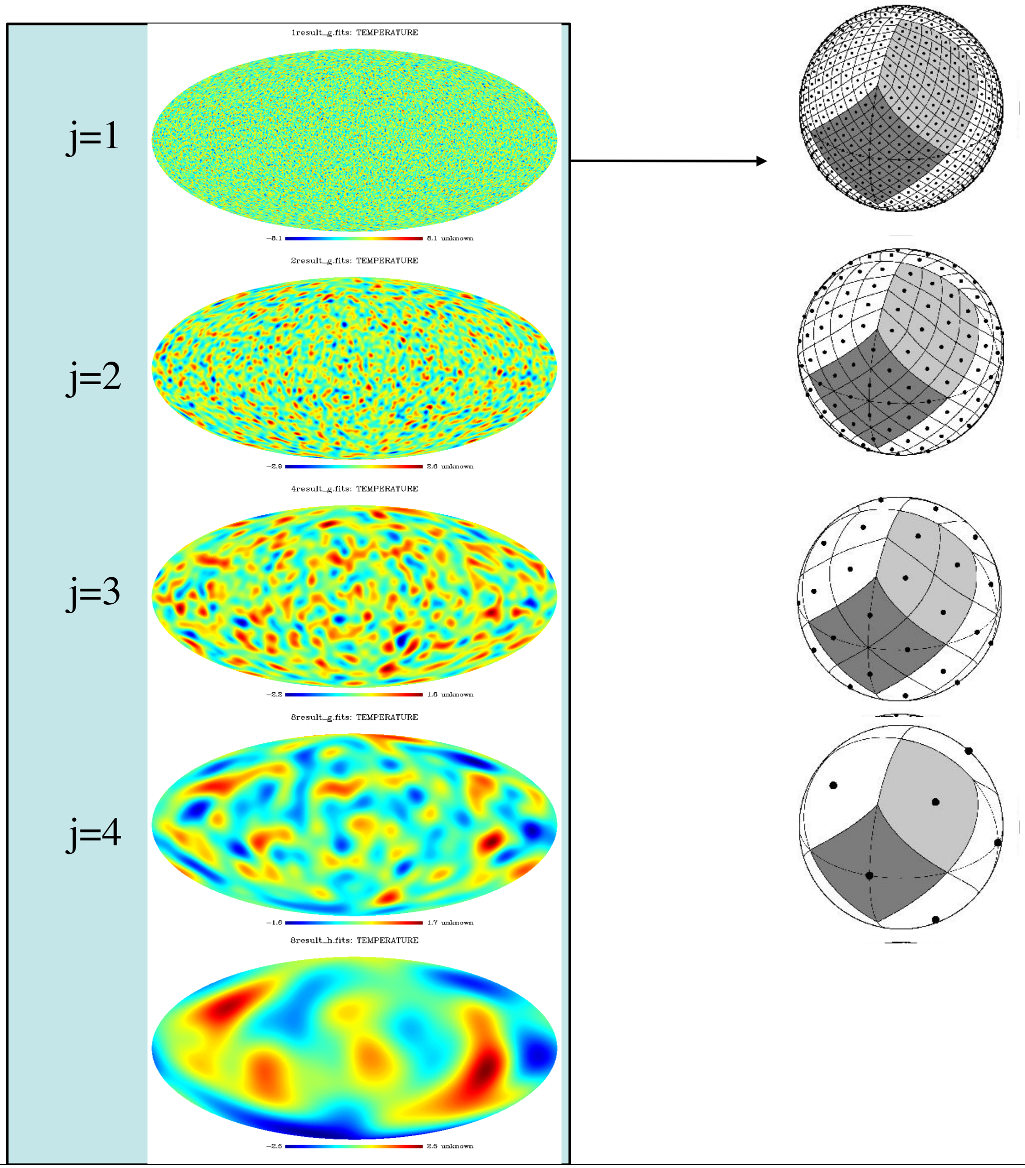


The sparse GMCA solution is obtained by minimizing:

$$\min_{\alpha_j, A_j} \sum_j \frac{1}{2\sigma^2} \left\| \beta_j - A_j \alpha_j \right\|^2 \quad \text{s.t.} \quad \alpha \quad \text{is sparse}$$

LOCAL GMCA

Undecimated Isotropic Wavelet Transform + Block Partitioning



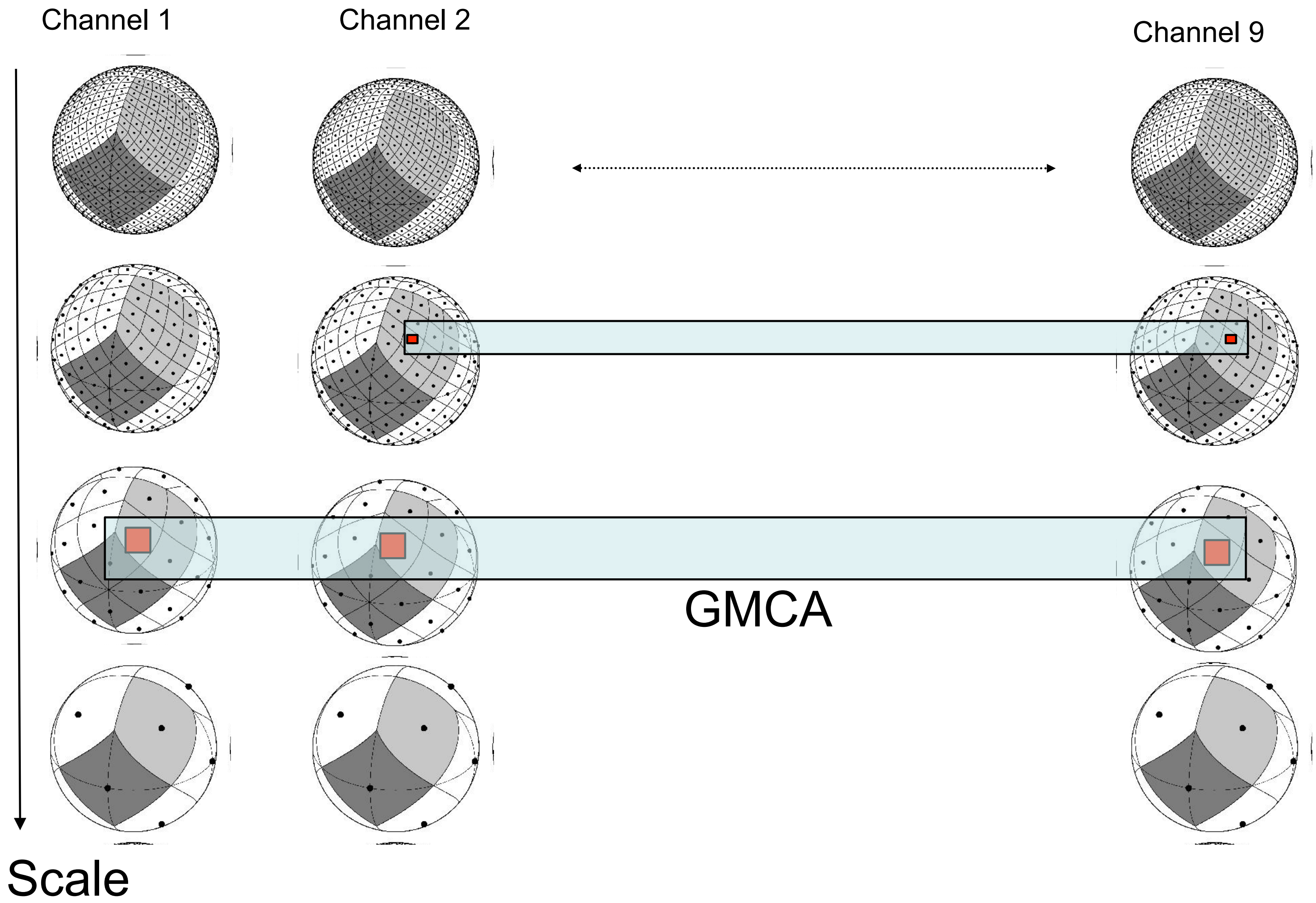
12*32 Blocks

12*16 Blocks

12*4 Blocks

12 Blocks

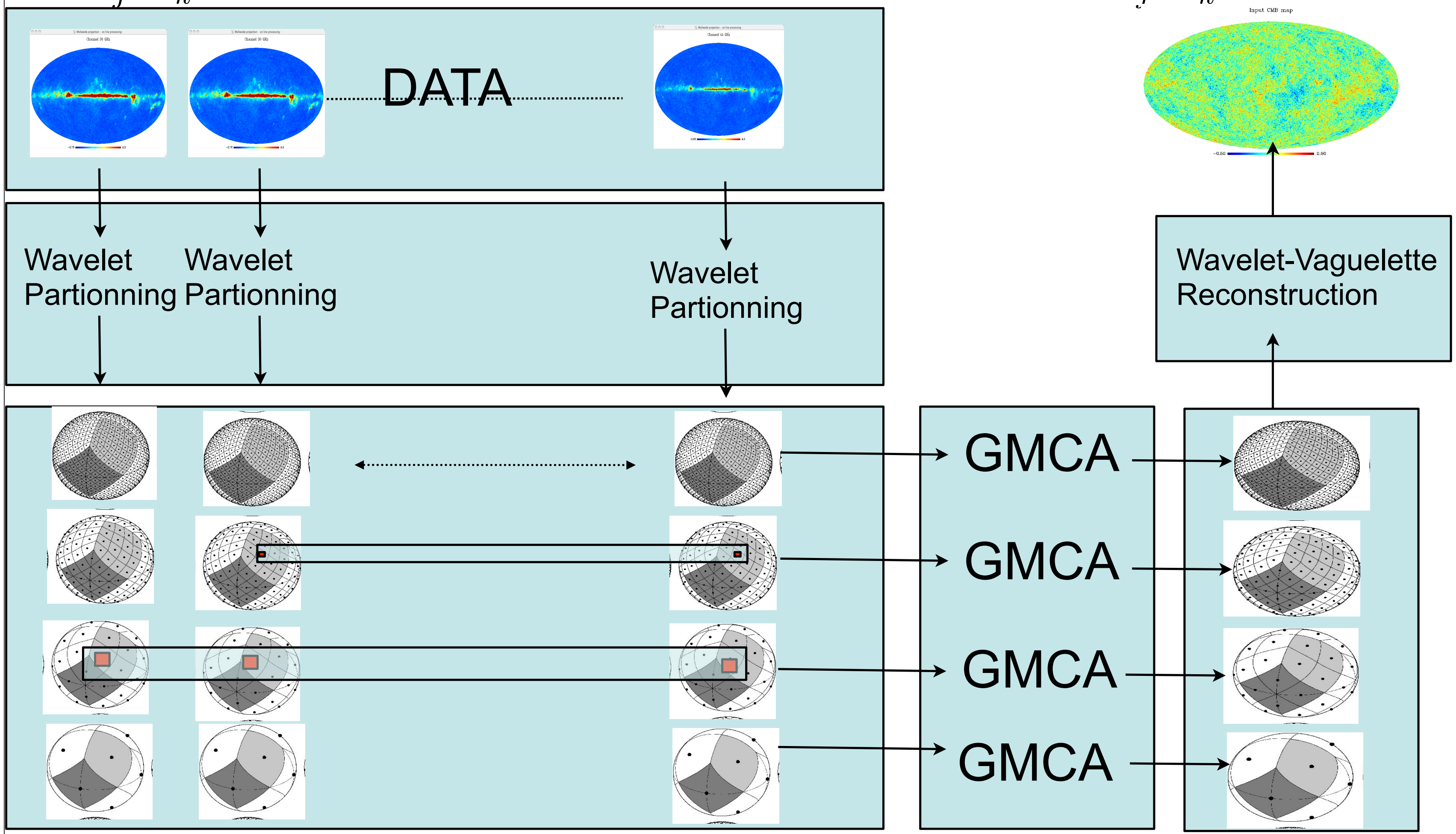
Global


Channel 1
Channel 2
Channel 9
GMCA
Scale

Wavelet-Vaguelette GMCA Decomposition

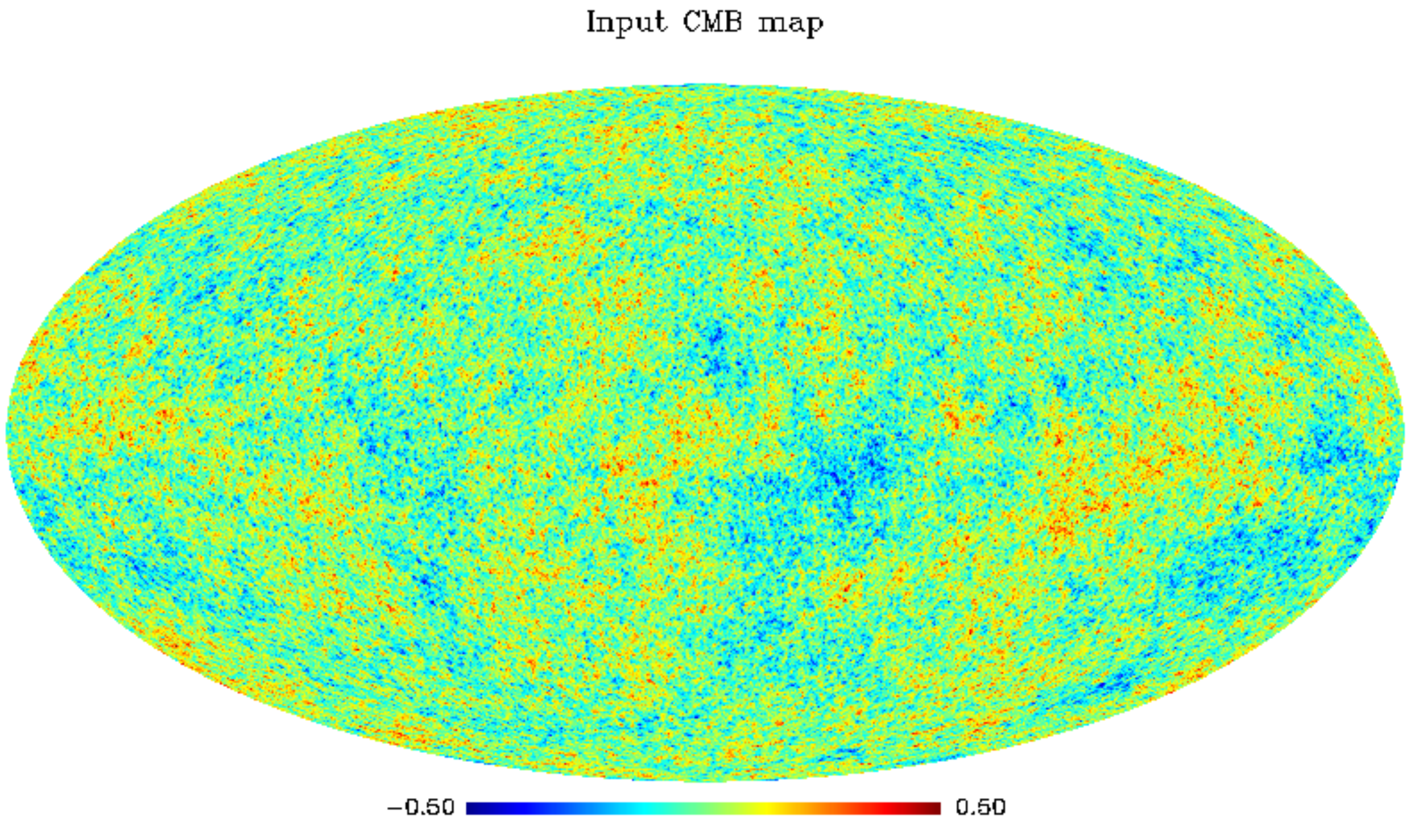
$$f = \sum_j \sum_k \langle Kf, \Psi_{j,k} \rangle \psi_j, k \quad \text{with } K^* \Psi_{j,k} = \psi_{j,k}$$

$$\tilde{f} = \sum_j \sum_k \Delta(\langle y, \Psi_{j,k} \rangle) \psi_j, k$$



CMB map estimation

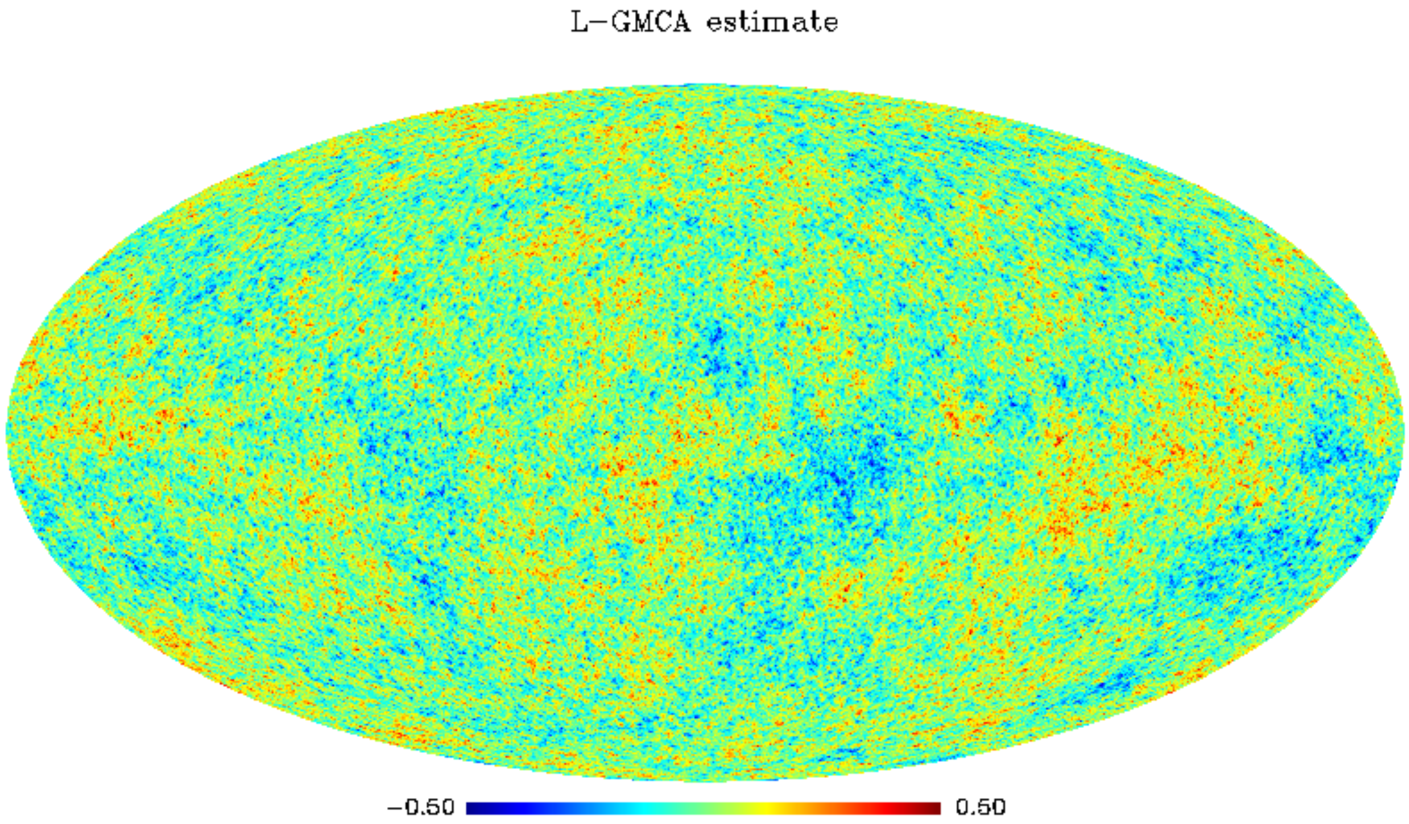
Input



Input CMB Map

CMB map estimation

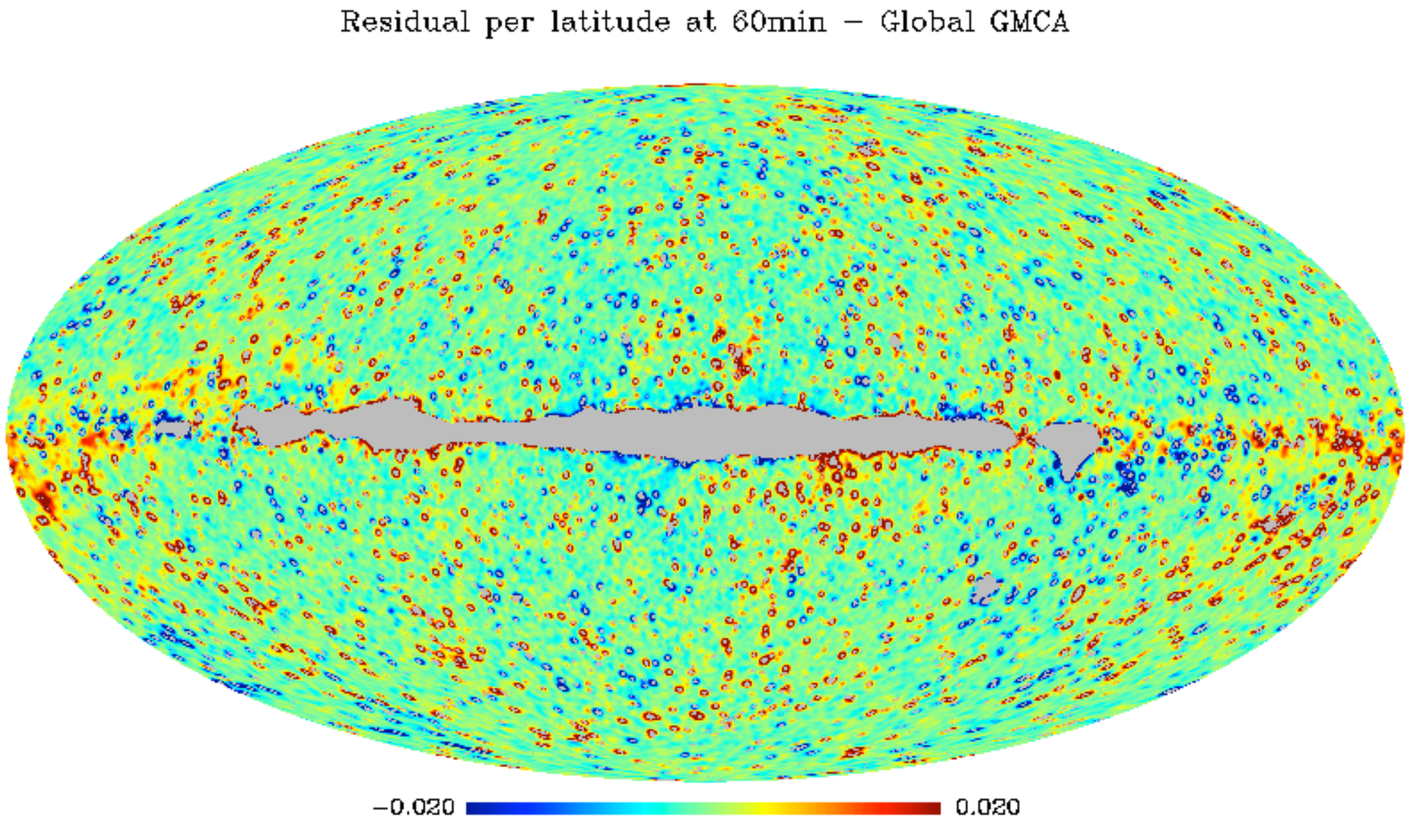
IGMCA



Estimate CMB Map

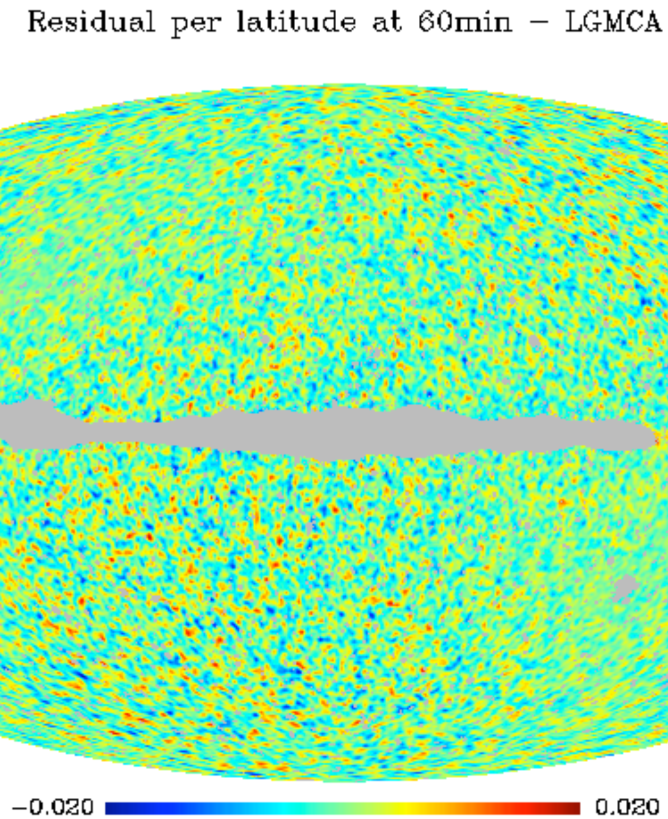
CMB map estimation

Residual at 60min



Residual map for Global GMCA

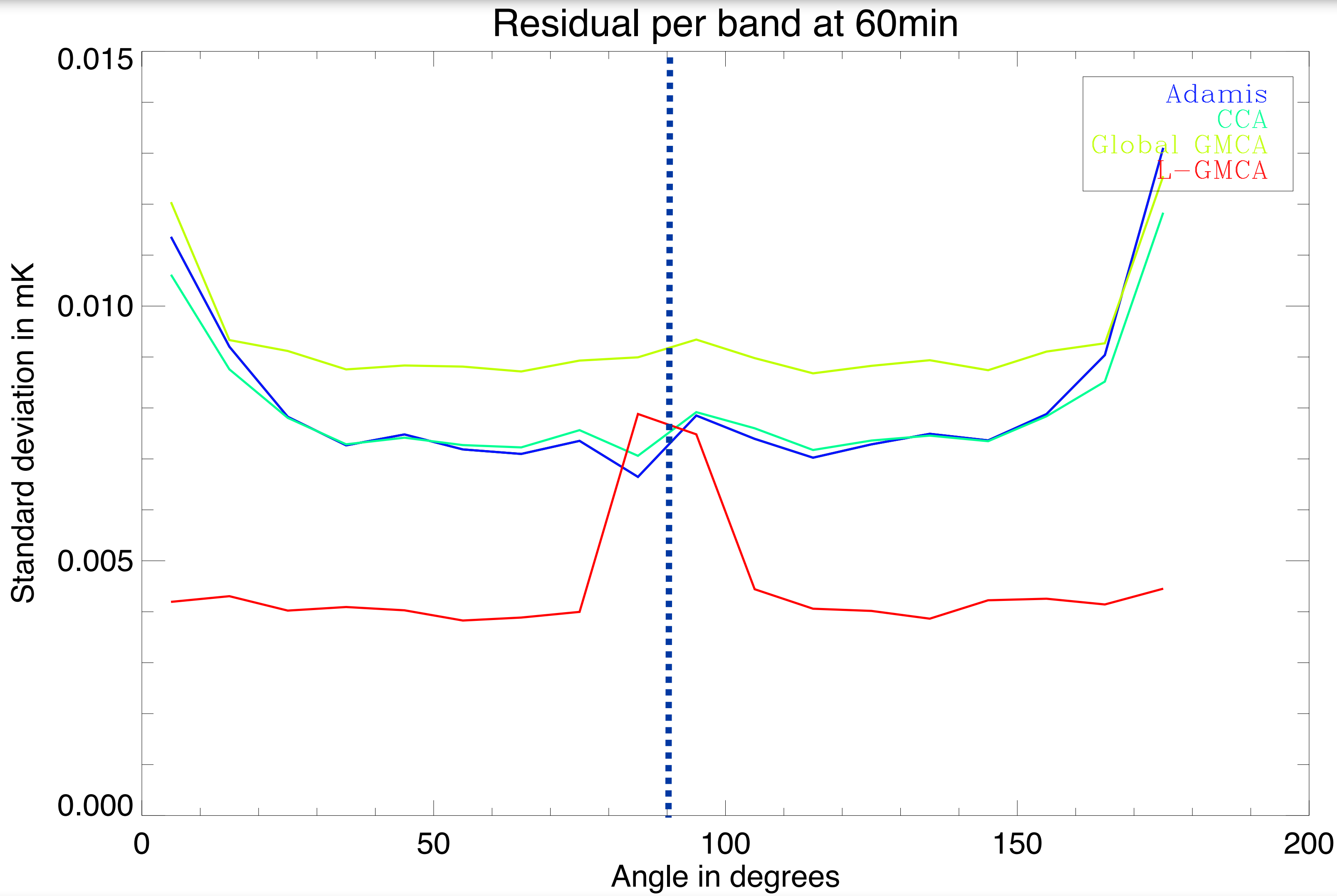
Residual at 60min



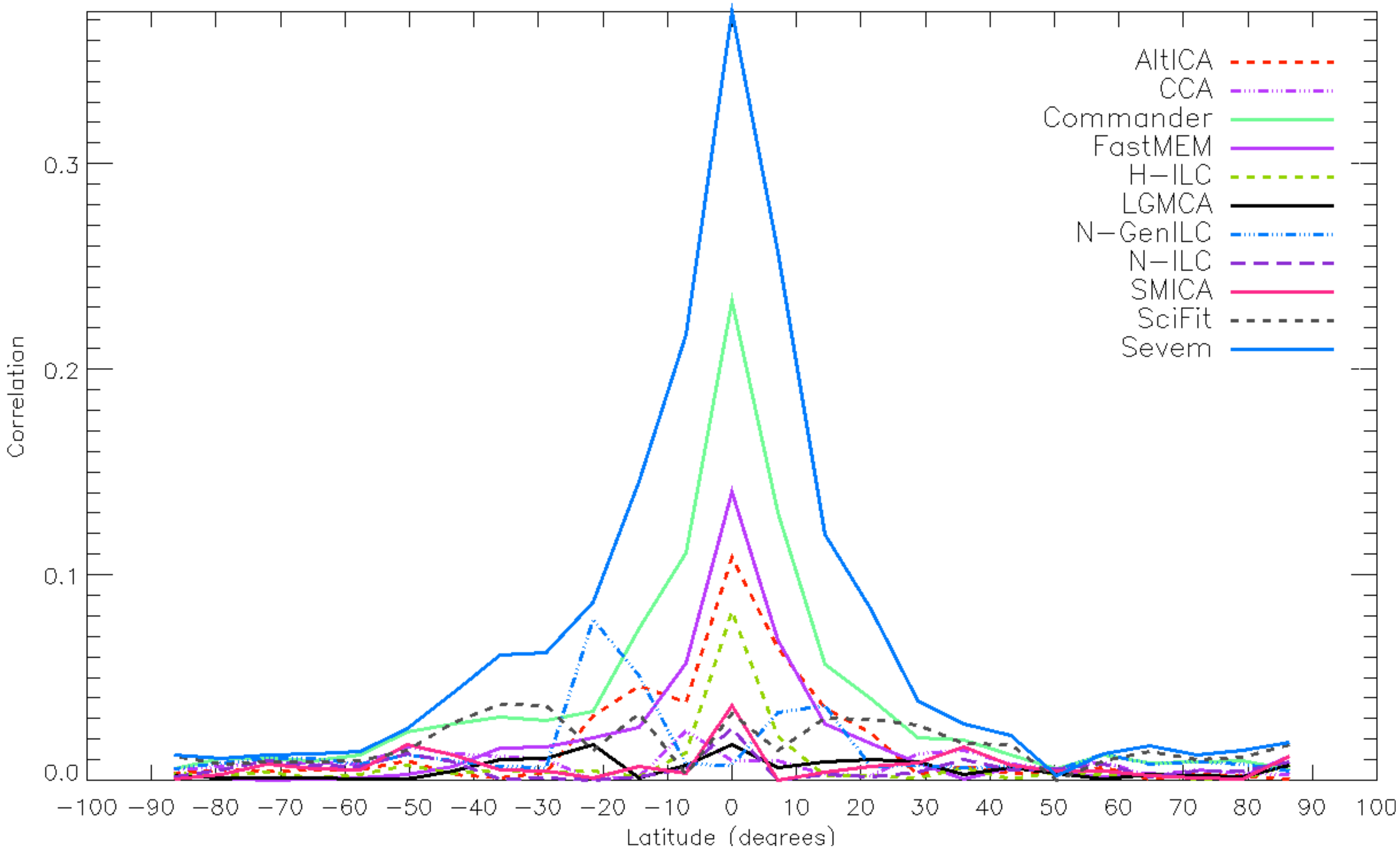
Residual map for L-GMCA

CMB map estimation

Residual per latitude



Cross-Correlation with Dust Template per Latitude Band: Wavelet Scale 1 (l=[1500, 3000])



Correlation of CMB Wavelet Scale 1 (l=2250) with Dust template (**IRIS map, Miville-Deschene & Lagache, 2005**).

Wiener Filtering

Noise have an additive contribution to the CMB map : $y = x + n$

Standard approach - Wiener in spherical harmonics: $\forall \ell, m > 0; \quad a_{\ell m}^{\hat{x}} = \frac{C_\ell}{C_\ell + C_\ell^n} a_{\ell m}^{y}$

But does not account for the non-stationarity of the noise !!

More rigorously,

$$\min_x \quad x^T \mathcal{F}^H \mathbf{C}^{-1} \mathcal{F} x + (y - x)^T \mathbf{\Sigma}^{-1} (y - x)$$

CMB power spectrum ($\mathbf{C}$)

Sph. Harmonics ($\mathcal{F}$)

Noise covariance matrix ($\mathbf{\Sigma}$)

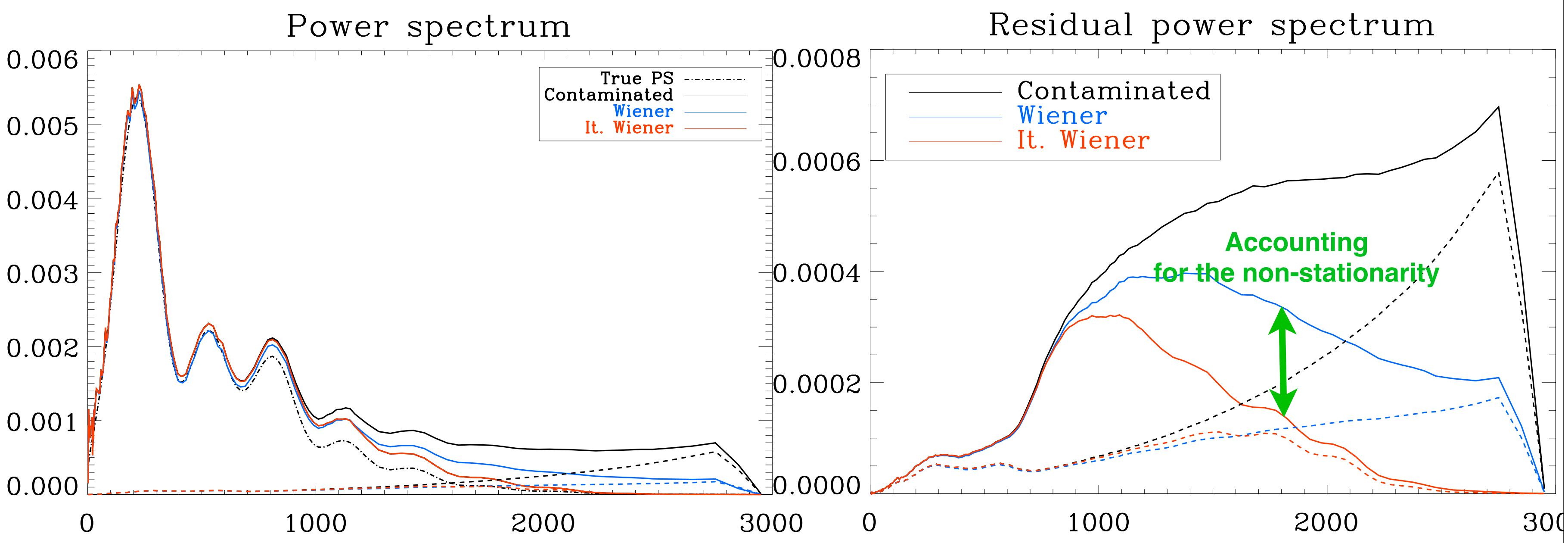
But wavelets are well known to decorrelate 1/f noise ==>

LInear Wavelet-based Filtering (Iterative Wiener Algorithm)

- J. Bobin, J.-L. Starck, F. Sureau, J. Fadili, "***CMB Map Restoration***, **http://arxiv.org/abs/1111.3149**, submitted.

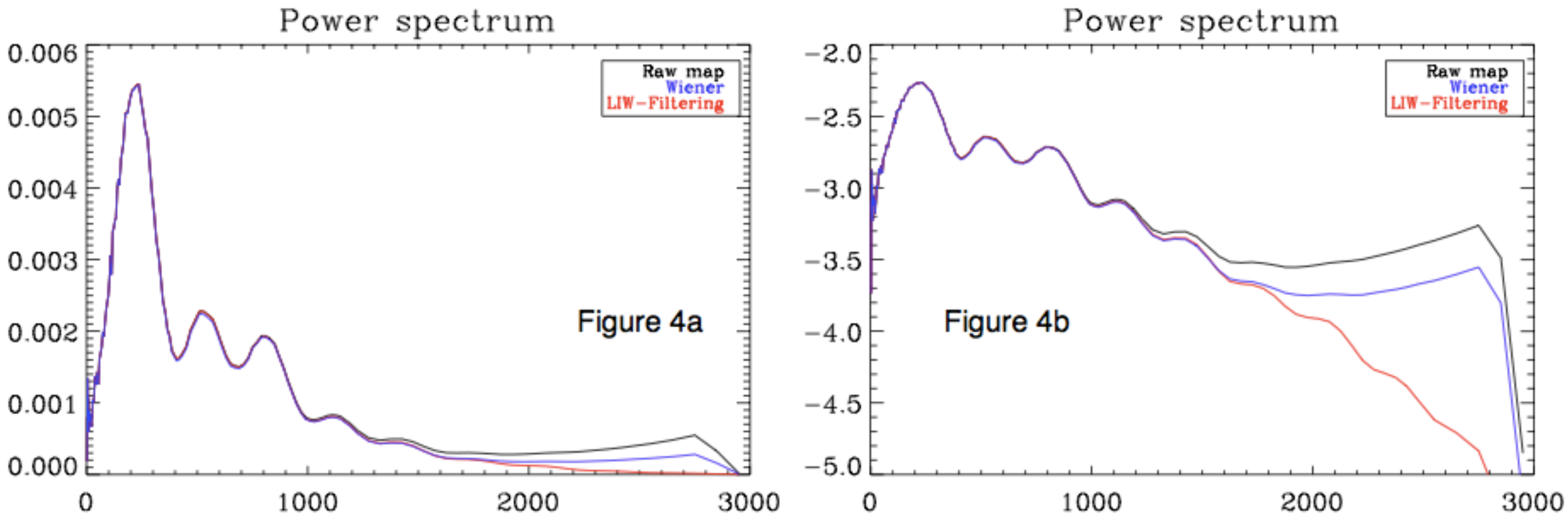
- Wavelet-based Gaussian modeling of the noise covariance matrix, either from the JackKnife map or noise simulations.
- Solve the equation using an iterative Forward-Backward Algorithm.

Result on WG2 Simulations



GMCA-LIW Filtering

CMB power spectrum of the CMB map computed by:
- GMCA CMB (black)
- GMCA + Wiener (blue)
- GMCA + LIW-filtering (in red).



Beyond Noise Reduction

Whatever the estimator used to estimate the CMB map, the CMB map will be contaminated by spurious foreground residuals

$$y = x + n + f$$

We use exactly the same wavelet-based Gaussian modeling.

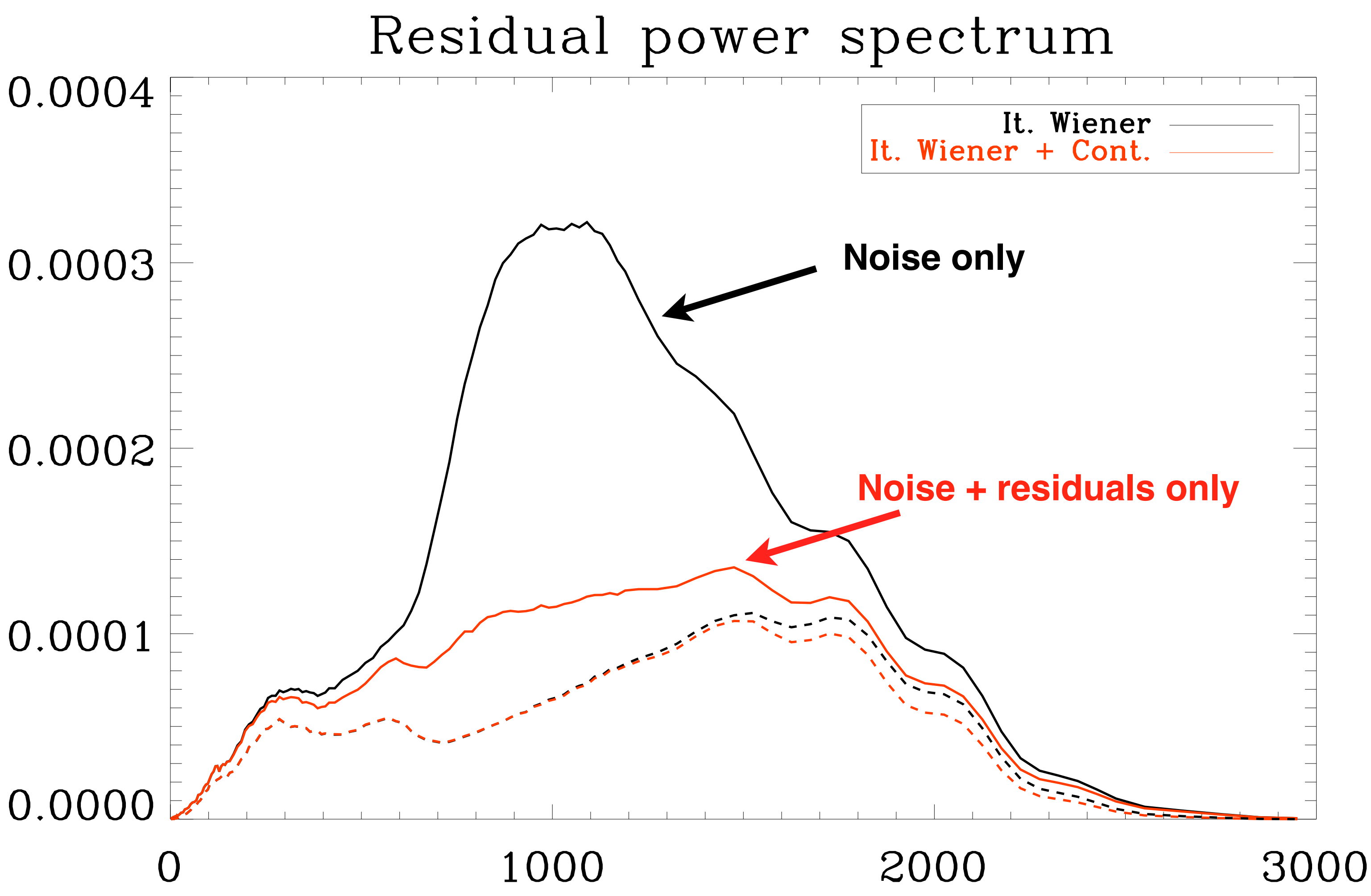
From the decorrelation of CMB, noise and foregrounds:

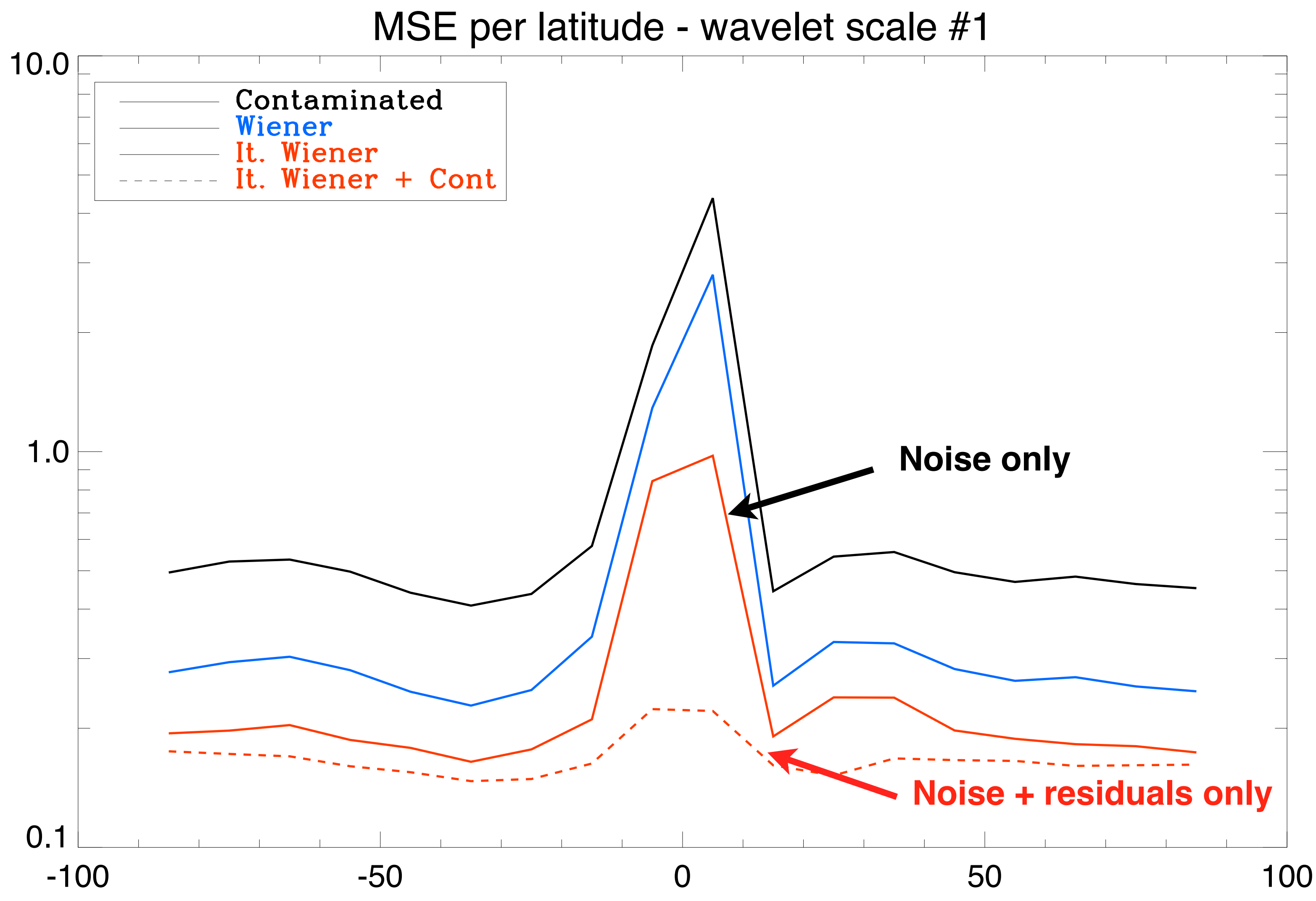
$$\hat{\sigma}_j^y[k]^2 = \sigma_j^2 + \hat{\sigma}_j^n[k]^2 + \hat{\sigma}_j^f[k]^2$$

CMB **Noise** **Foregrounds**

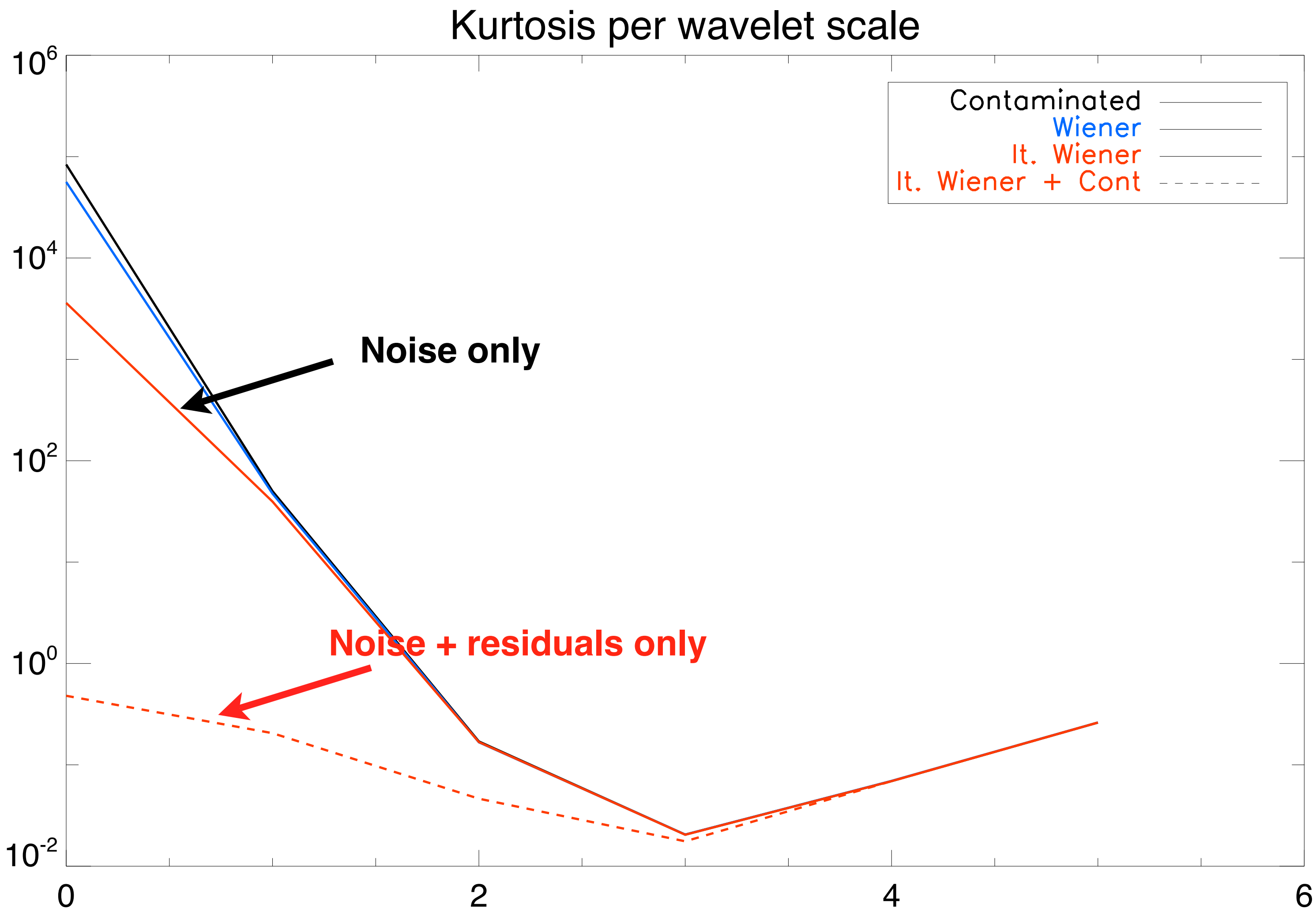
$$\hat{\sigma}_j^f[k]^2 = \left[\hat{\sigma}_j^y[k]^2 - \frac{1}{4\pi} \sum_\ell (2\ell+1) \psi_{j,\ell}^2 C_\ell - \hat{\sigma}_j^n[k]^2 \right]_+$$

Some results

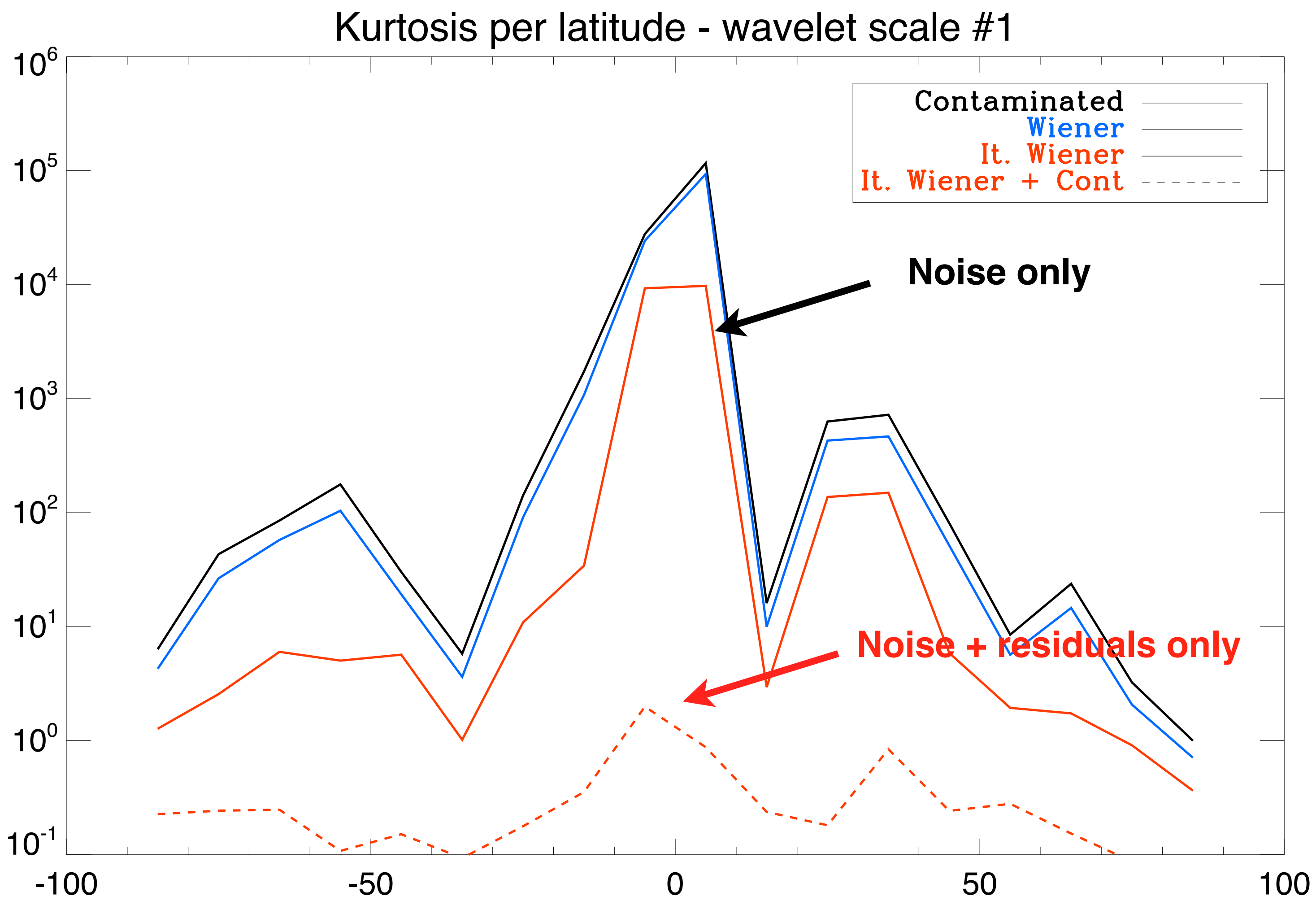



MSE per latitude - wavelet scale #1
10.0
1.0
0.1
-100
-50
0
50
100
Contaminated
Wiener
It. Wiener
It. Wiener + Cont
Noise only
Noise + residuals only

Some results



Some results



Sparsity and CMB

- Sparsity is very efficient for
  - Inverse problems (denoising, deconvolution, etc).
  - Inpainting
  - Component Separation.
  - Wiener Wiltering.

- Perspectives
  - Estimator Aggregation.
  - Dictionary Learning.

Postdoc position available at CEA-Saclay