Efficiently Testing Simon's Congruence

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Tore Koß Stefan Siemer

Combinatorics on Words Online Seminar
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Subsequences

w $\boxed{b \quad a \quad c \quad b \quad a \quad a \quad b \quad a \quad d \quad a}$

Subsequences

ababa

d

bbb

ba

bc

w $\boxed{b \quad a \quad c \quad b \quad a \quad a \quad b \quad a \quad d \quad a}$

baab

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w	b	a	c	b	a	a	b	a	d	a

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Subsequences

Subsequence

We call w' a subsequence of length k of a word w, where $|w| = n$, if there exist positions $1 \leq i_1 < i_2 < \ldots < i_k \leq n$, such that $w' = w[i_1]w[i_2]\cdots w[i_k]$.

Set of Subsequences of length k

Let $\mathrm{Subseq}_k(i, w)$ denote the set of subsequences of length k of $w[i : n]$. Accordingly, the set of subsequences of length k of the entire word w will be denoted by $\mathrm{Subseq}_k(1, w)$.

Example: $\mathrm{Subseq}_2(1, abaca) = \{aa, ab, ac, ba, bc, ca\}$

Simon's Congruence

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(i) Let $w, w' \in \Sigma^*$. We say that w and w' are equivalent under Simon's congruence $\sim_k$ if $\mathsf{Subseq}_k(1, w) = \mathsf{Subseq}_k(1, w')$.

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$$\mathrm{Subseq}_2(1, w') = \{aa, ab, ac, ba, bb, bc, ca, cb\}$$
$$\mathrm{Subseq}_2(1, w) = \mathrm{Subseq}_2(1, w') \Rightarrow w \sim_2 w'$$

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$$bbb \notin \mathsf{Subseq}_3(1, w), bbb \in \mathsf{Subseq}_3(1, w')$$
$$\mathsf{Subseq}_3(1, w) \neq \mathsf{Subseq}_3(1, w') \Rightarrow w \nsim_3 w'$$

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(ii) Let $1 \leq i < j \leq |w|$. We define $i \sim_k j$ (w.r.t. w) if $w[i:n] \sim_k w[j:n]$, and we say that the positions i and j are k-equivalent.

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(iii) A word u of length k distinguishes w and w' w.r.t. $\sim_k$ if u occurs in exactly one of the sets $\mathsf{Subseq}_k(1, w)$ and $\mathsf{Subseq}_k(1, w')$.

Problem Definition

SimK

Given two words s and t over an alphabet Σ, with $|s| = n$ and $|t| = n'$, with $n \geq n'$, and a natural number k, decide whether $s \sim_k t$.

MaxSimK

Given two words s and t over an alphabet Σ, with $|s| = n$ and $|t| = n'$, with $n \geq n'$, find the maximum k for which $s \sim_k t$.

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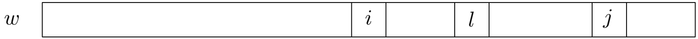
- Today: the first optimal linear-time algorithm for the MaxSimK problem. [STACS 2021]

Simon-tree

Equivalence Classes



$$\mathcal{SF}_k(i,w) \supset \mathcal{SF}_k(l,w) \supset \mathcal{SF}_k(j,w)$$

- Splitting a word suffixwise into blocks of equivalence classes w.r.t. $\sim_k$
- If $i \sim_k j$, then $\mathrm{Subseq}_k(i,w) = \mathrm{Subseq}_k(l,w) = \mathrm{Subseq}_k(j,w)$ and we say that i, l, and j are in the same *k-block*
- $\sim_{k+1}$ is a refinement of $\sim_k$
- Index i is a $(k+1)$*-splitting position* if $i \sim_k i+1$ but not $i \sim_{k+1} i+1$

Equivalence Classes

Use these properties to build a block structure for a word

1. $i \sim_1 j$ iff $\mathsf{alph}(w[i:n]) = \mathsf{alph}(w[j:n])$ for any $1 \leq i < j \leq |w|$
   $\rightarrow$ We can go from right to left through the word and determine 1-splitting positions

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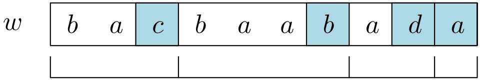

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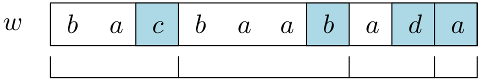

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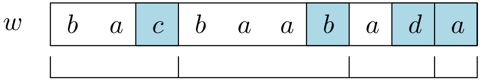

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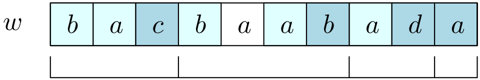

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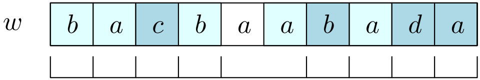

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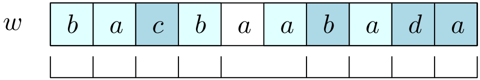

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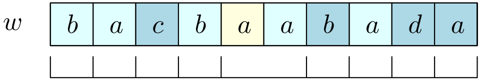

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w	b	a	c	b	a	a	b	a	d	a

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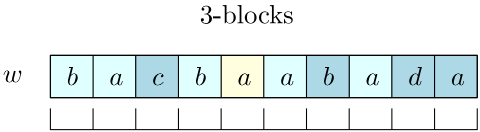
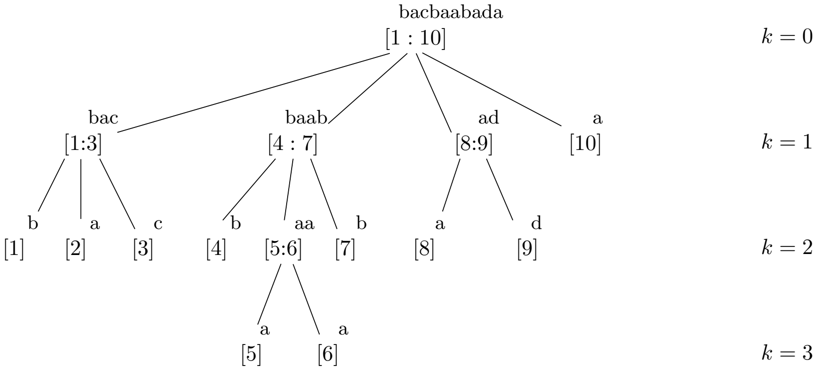


Simon-tree Definition

- New data structure: Simon-tree
- Represents presented block structure
- Efficiently partition positions of a given word
- Construction takes linear time

position	1	2	3	4	5	6	7	8	9	10	11
w	b	a	c	b	a	a	b	a	d	a	$
X	4	5	∞	7	6	8	∞	10	∞	∞	∞



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Simon-tree

The *Simon-tree* T_w associated to the word w, with $|w| = n$, is an ordered rooted tree. The nodes represent $k-$blocks of w, for $0 \leq k \leq n$, and are defined recursively.

- The root corresponds to the 0-block of the word w, i.e., the interval $[1 : n]$.
- For $k > 1$ and for a node b on level $k - 1$, which represents a $(k-1)$-block $[i : j]$ with $i < j$, the children of b are exactly the blocks of the partition of $[i : j]$ in k-blocks, ordered decreasingly by their starting position.
- For $k > 1$, each node on the level $k - 1$ which represents a $(k-1)$-block $[i : i]$ is a leaf.

Simon-tree Construction

- **Algorithm:** Build the Simon-tree right to left as the word is traversed right to left. Only the leftmost branch is edited during construction.
    1. Insert the new position/letter into the tree by moving up the leftmost branch from leaf to root.
    2. Find lowest node that is not split by this position (and close all the others on the way).
    3. Insert the new position/letter as a leftmost child of this node.

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$$\begin{array}{c} \ldots\$ \\ 11] \end{array}$$

$k = 0$

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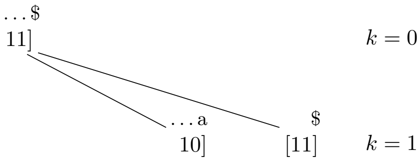

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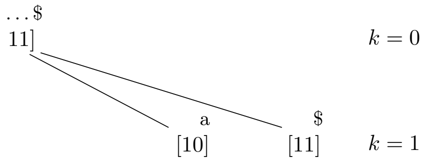

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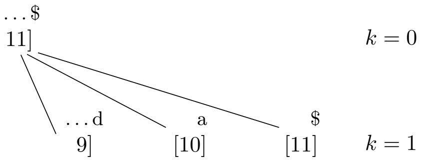

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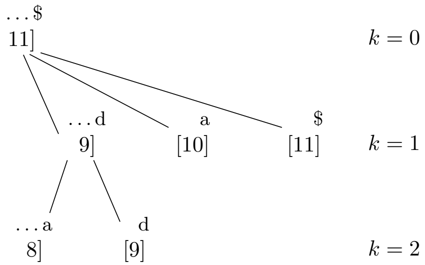

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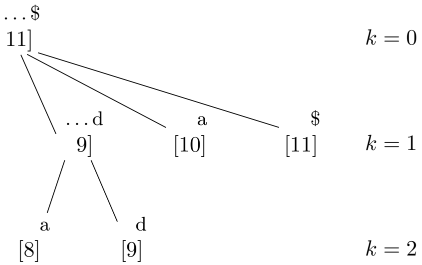

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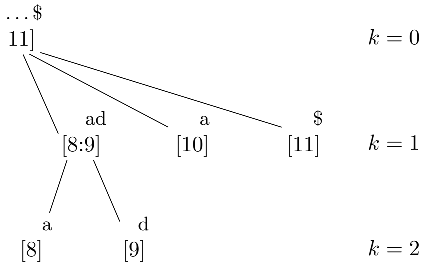

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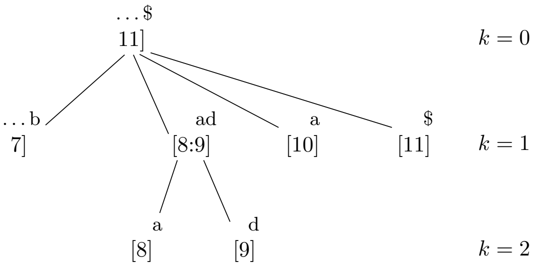

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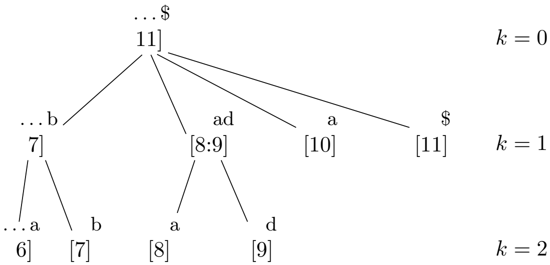

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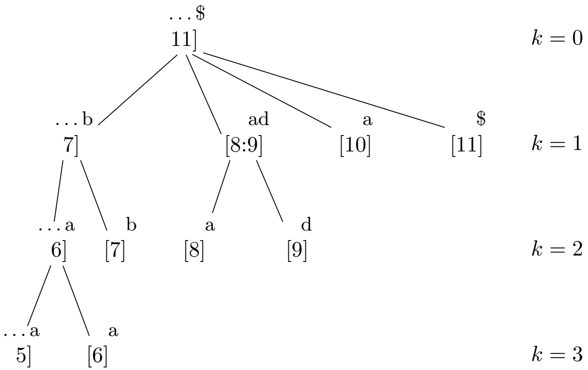

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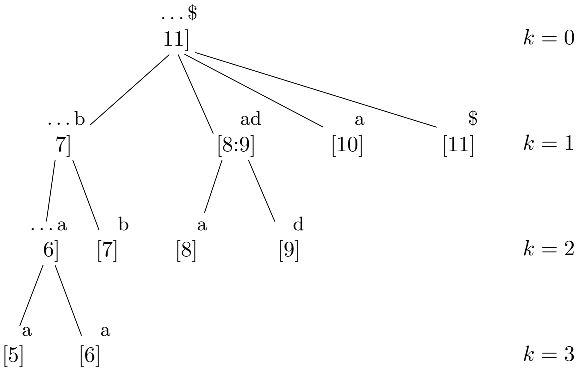

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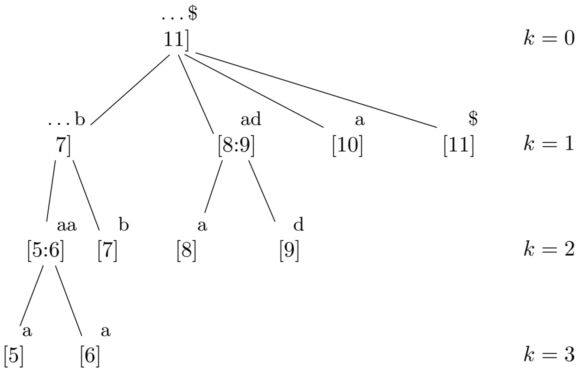

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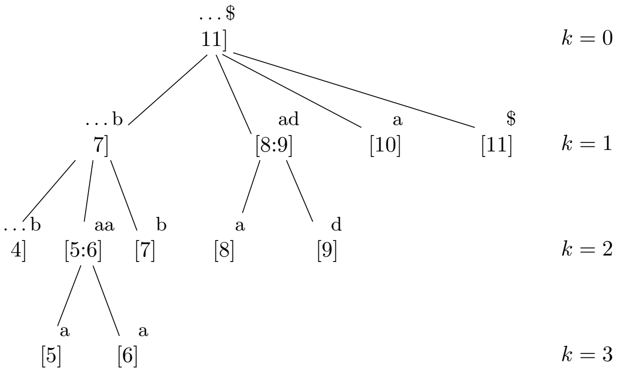
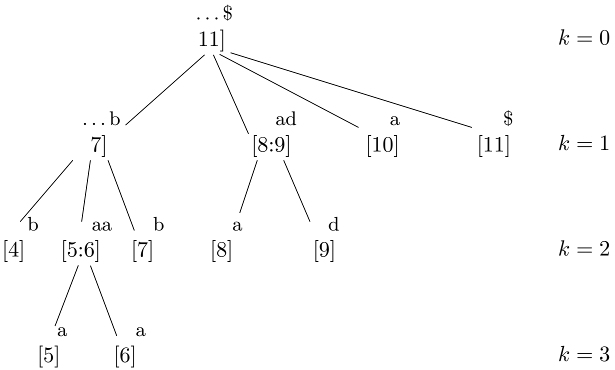


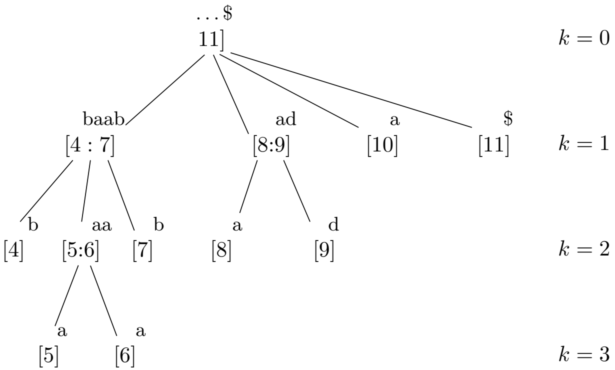
Simon-tree Construction

position	1	2	3	4	5	6	7	8	9	10	11
w	b	a	c	b	a	a	b	a	d	a	\$
X	4	5	∞	7	6	8	∞	10	∞	∞	∞



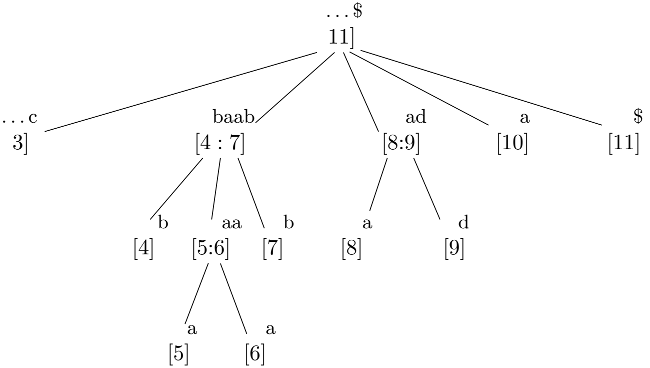
Simon-tree Construction

position	1	2	3	4	5	6	7	8	9	10	11
w	b	a	c	b	a	a	b	a	d	a	\$
X	4	5	∞	7	6	8	∞	10	∞	∞	∞



Simon-tree Construction

position	1	2	3	4	5	6	7	8	9	10	11
w	b	a	c	b	a	a	b	a	d	a	$
X	4	5	∞	7	6	8	∞	10	∞	∞	∞



$k = 0$

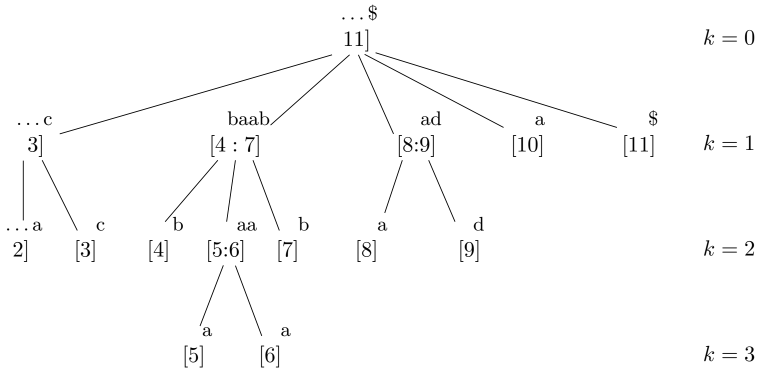
$k = 1$

$k = 2$

$k = 3$

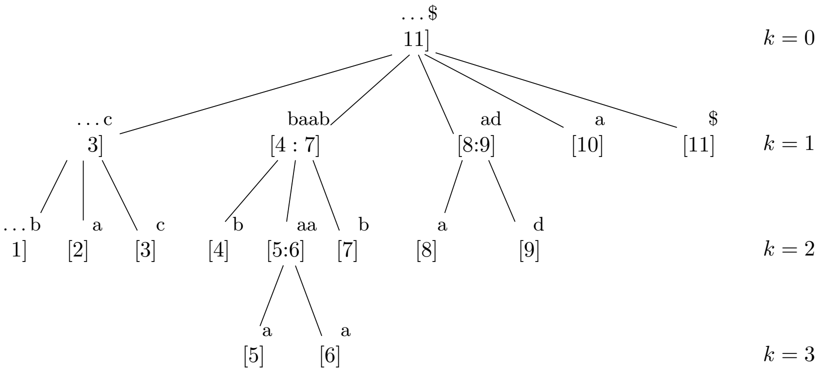
Simon-tree Construction

position	1	2	3	4	5	6	7	8	9	10	11
w	b	a	c	b	a	a	b	a	d	a	\$
X	4	5	∞	7	6	8	∞	10	∞	∞	∞



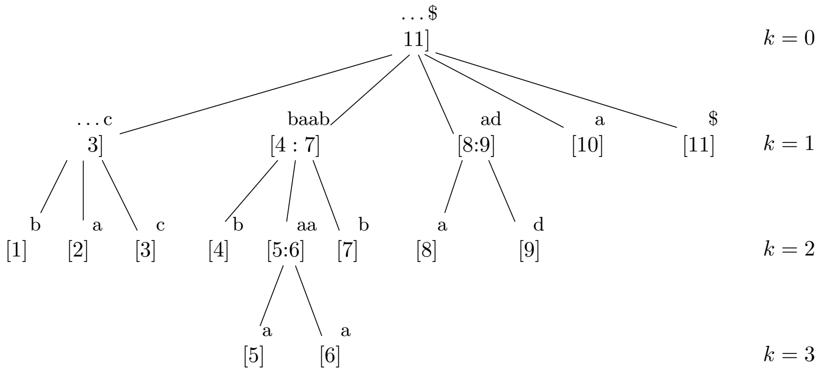
Simon-tree Construction

position	1	2	3	4	5	6	7	8	9	10	11
w	b	a	c	b	a	a	b	a	d	a	$
X	4	5	∞	7	6	8	∞	10	∞	∞	∞



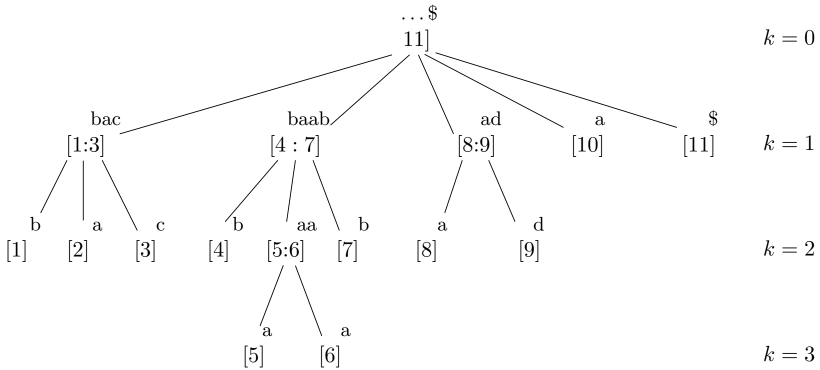
Simon-tree Construction

position	1	2	3	4	5	6	7	8	9	10	11
w	b	a	c	b	a	a	b	a	d	a	\$
X	4	5	∞	7	6	8	∞	10	∞	∞	∞



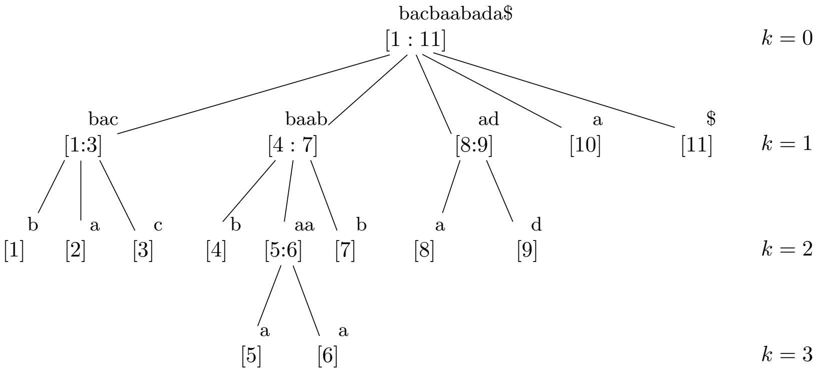
Simon-tree Construction

position	1	2	3	4	5	6	7	8	9	10	11
w	b	a	c	b	a	a	b	a	d	a	$
X	4	5	∞	7	6	8	∞	10	∞	∞	∞



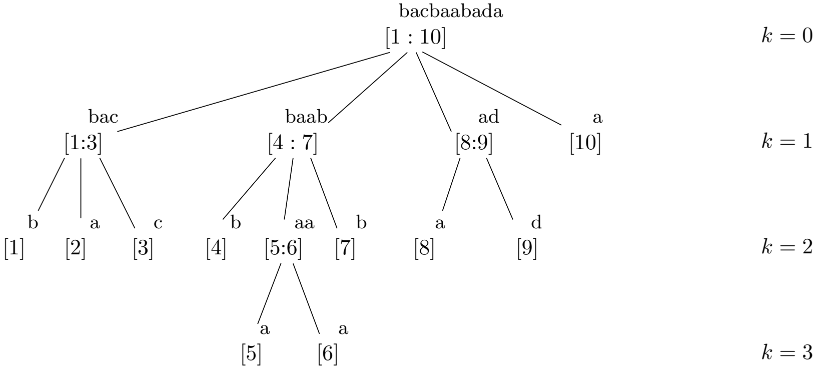
Simon-tree Construction

position	1	2	3	4	5	6	7	8	9	10	11
w	b	a	c	b	a	a	b	a	d	a	$
X	4	5	∞	7	6	8	∞	10	∞	∞	∞



Simon-tree Construction

position	1	2	3	4	5	6	7	8	9	10	11
w	b	a	c	b	a	a	b	a	d	a	\$
X	4	5	∞	7	6	8	∞	10	∞	∞	∞



Simon-tree Construction

- **Algorithm:** Build the Simon-tree right to left as the word is traversed right to left. Only the leftmost branch is edited during construction.
- **Complexity:** linear! All nodes appear only once on the leftmost path, until they are closed.

Short Recap

So far:
structure for one word representing the equivalence classes w.r.t. $\sim_k$

MaxSimK
Given two words s and t over an alphabet Σ, with $|s| = n$ and $|t| = n'$, with $n \geq n'$, find the maximum k for which $s \sim_k t$.

Now:
set two words in relation to each other by using their respective Simon-trees

Connecting Two Simon-trees

- Transform the words s and t into Simon-trees as shown.
- Use the tree structure to connect equivalent nodes of the two words.

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S-Connection

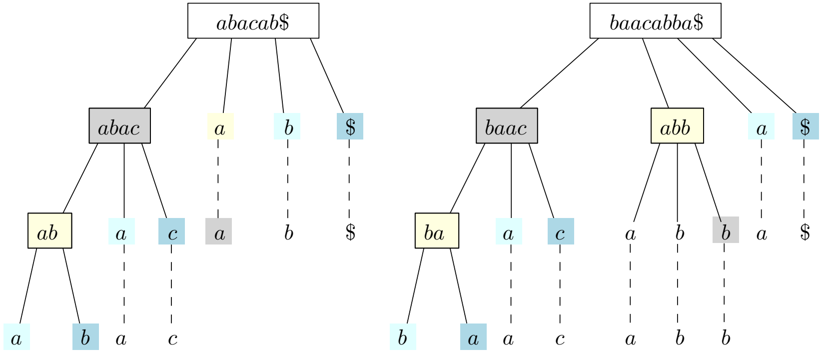
The k-node a of T_s and the k-node b of T_t are S-connected (i.e., the pair (a, b) is in the S-connection) if and only if $s[i:n] \sim_k t[j:n']$ for all positions i in block a and positions j in block b.

From P-Connection to S-Connection

Starting from a larger relation (P-Connection) which contains the S-Connection, and refine it.

- The 0-nodes of T_s and T_t are P-connected.
- For all levels k of T_s, if the explicit or implicit k-nodes a and b (from T_s and T_t, respectively) are P-connected, then the i^{th} child of a is P-connected to the i^{th} child of b, for all i.
- No other nodes are P-connected.

From P-Connection to S-Connection

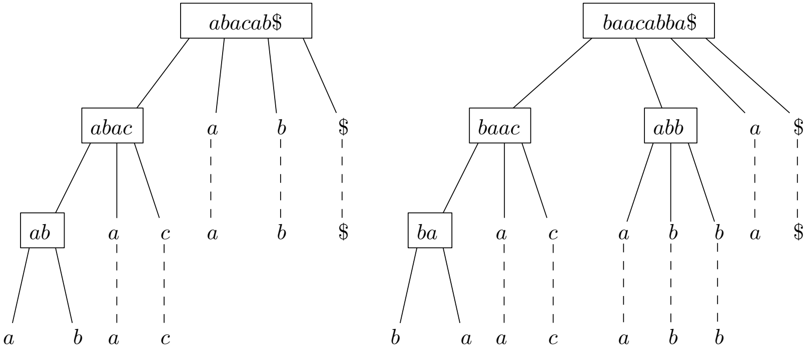


From P-Connection to S-Connection

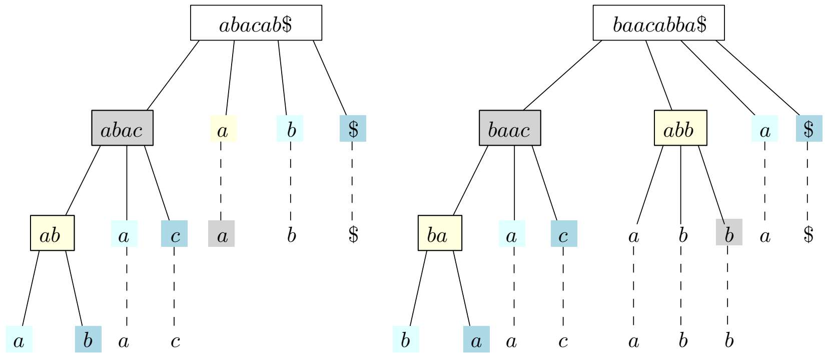
How to refine the P-Connection:

- Let $k \geq 1$ and a, b be k-blocks in the word t, resp. s, which are S-connected.
- Let a' be child of a, b' be child of b.
- $a' \sim_{k+1} b'$ if and only if there exists a letter x such that $s[\mathtt{next}(a', x) + 1 : n] \sim_k t[\mathtt{next}(b', x) + 1 : n']$.

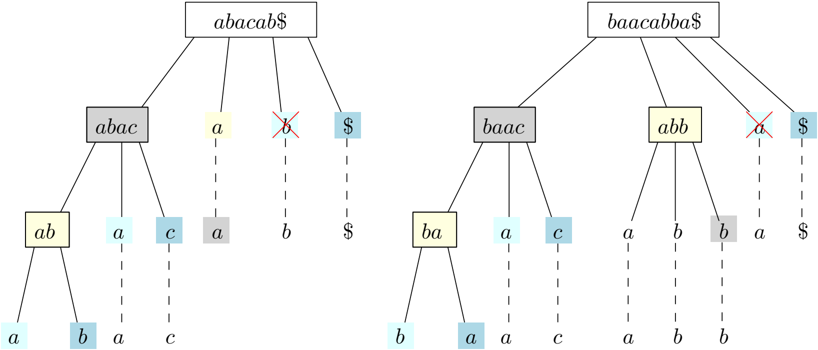
From P-Connection to S-Connection



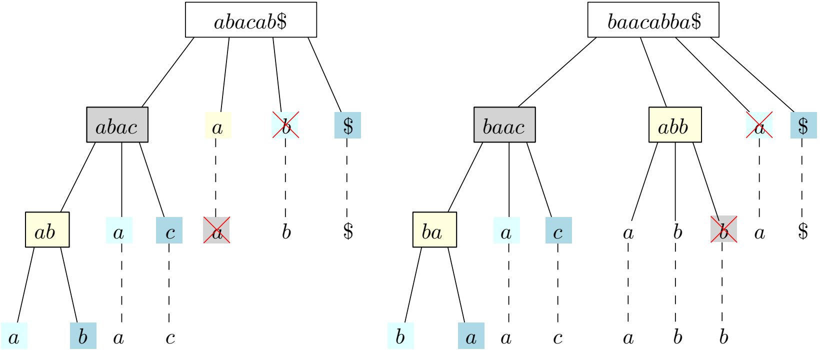
From P-Connection to S-Connection



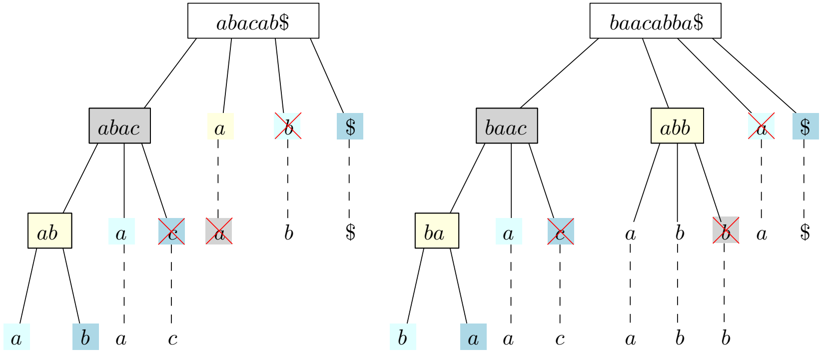
From P-Connection to S-Connection



From P-Connection to S-Connection



From P-Connection to S-Connection



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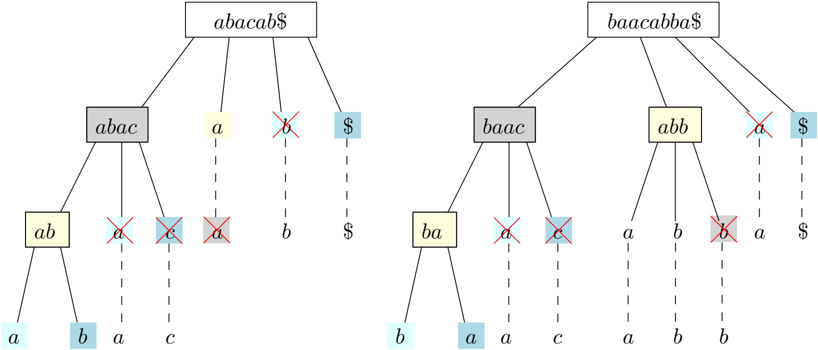
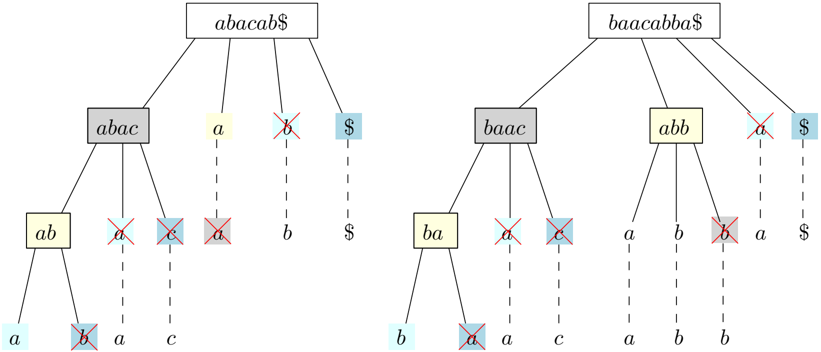
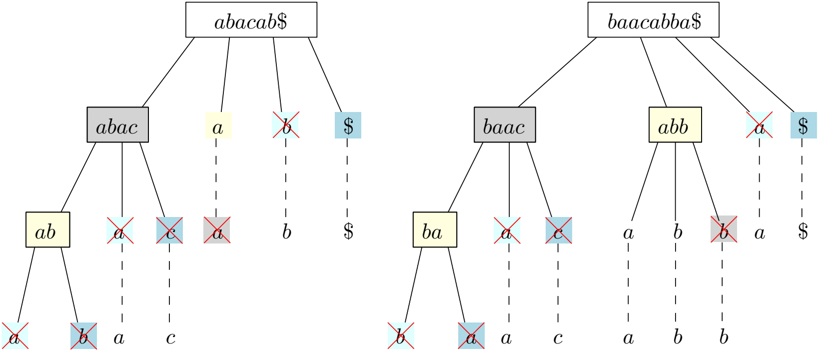


From P-Connection to S-Connection



From P-Connection to S-Connection



Additional Notes and Analysis

- Solution of MaxSimK: last level k where the k-blocks containing position 1 of the input words are equivalent.
- Distinguishing word can be obtained.
- By efficiently using interval-union-find and -split-find data structures the algorithm achieves an optimal linear runtime.

Theorem

MaxSimK*can be solved in optimal linear time.*

The End

Future Work

- Edit/Hamming Distance to $\sim_k$-equivalence.
  First steps: [Day, Fleischmann, Kosche, Tore Koß, M., Siemer: The Edit Distance to k-Subsequence Universality, STACS'21]

The End

Future Work

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Thank You!