Exponents of words under injective morphisms

Aleksi Vanhatalo[1]
based on
joint work with Eva Foster[2] and Aleksi Saarela[1],
Saarela's earlier work and
additional on-going work.

[1]Department of Mathematics and Statistics
University of Turku, Finland

[2]School of Computing and Mathematical Sciences, Birkbeck College
University of London, United Kingdom

One World Combinatorics on Words, 2025

What we want to investigate?

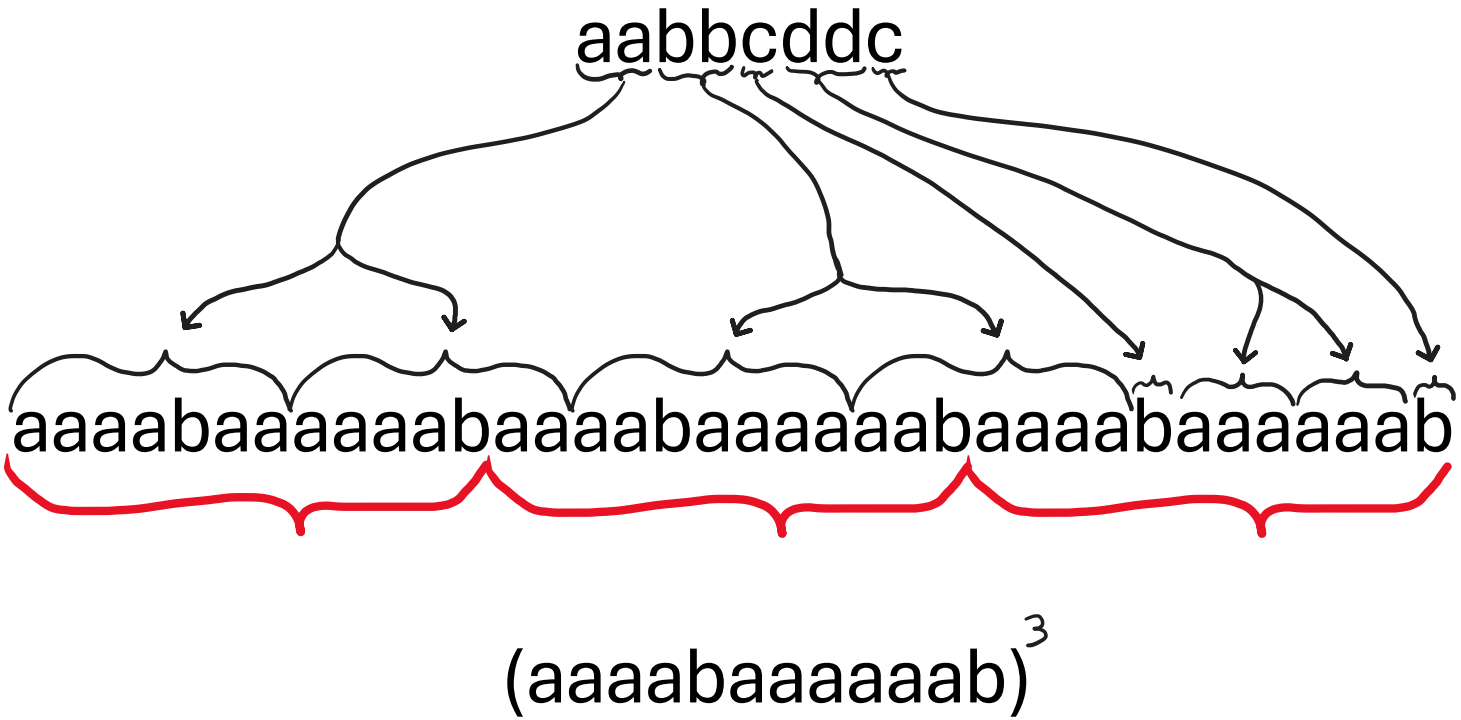

Figure: An injective morphism mapping primitive word to a cube. Example by Saarela (2025).

Table of Contents

The basics

- Let $v \in \Sigma^+$, $n \in \mathbb{N}$ and $r = \frac{n}{|v|} \in \mathbb{Q}$.
- Then by v^r we denote the prefix of length n of the infinite word $vvv\cdots$.
- For example, $(aba)^{\frac{7}{3}} = abaabaa$.
- Morphisms are maps with the rule $h(uv) = h(u)h(v)$. Thus morphisms are defined by the images of letters.
- Injectivity of morphism means injectivity on finite words. I will ignore the fact that it is sometimes hard to check injectivity.
- For binary morphism h, h is injective iff $\{h(a), h(b)\} \not\subset v^*$ for any v (defect effect).

Exponents of words

Definition (The (fractional) exponent and the integer exponent)

Let $v \in \Sigma^*$. Then it has two exponent values:

$$\mathrm{E}(v) = \max\{r \in \mathbb{Q} \mid \exists u \in \Sigma^+ : v = u^r\},$$
$$\mathrm{IE}(v) = \max\{n \in \mathbb{N} \mid \exists u \in \Sigma^+ : v = u^n\}.$$

Definition (Critical exponent and asymptotic critical exponent)

Let $w \in \Sigma^{\mathbb{N}}$. Then it has two exponent values:

$$\mathrm{CE}(w) = \sup\{\mathrm{E}(v) \mid v \in \mathrm{Fact}_+(w)\},$$
$$\mathrm{ACE}(w) = \limsup_{n\to\infty}\{\mathrm{E}(v) \mid v \in \mathrm{Fact}_n(w)\}.$$

Earlier results on Asymptotic Critical Exponent

Theorem (R. Entringer, D. Jackson, and J. Schatz, 1974)

There exists an infinite binary word having no squares of length > 4.

Theorem (J. Beck, 1984 (or J. Cassaigne, 2008))

There exists an (effectively constructed) infinite binary word u *with* $\mathrm{ACE}(u) = 1$.

Our point of interest

We ask the following:

Our point of interest

We ask the following:

- Given a word, can we design a morphism that maps the word to a large repetition? **Which words have an upper bound on the exponent(s)** after mapping via morphisms?

Our point of interest

We ask the following:

- Given a word, can we design a morphism that maps the word to a large repetition? **Which words have an upper bound on the exponent(s)** after mapping via morphisms?
- These questions are trivial if the morphisms are not restricted in any way, since we can map all words to unitary alphabet. However, if we focus on **injective morphisms**, these questions become interesting.

New "exponents"

Definition (Mapped exponents)

$$\mathrm{E}_{\mathcal{I}}(u) = \sup\{\mathrm{E}(h(u)) \mid h \in \mathcal{I}\},$$
$$\mathrm{IE}_{\mathcal{I}}(u) = \sup\{\mathrm{IE}(h(u)) \mid h \in \mathcal{I}\} \text{ [Saarela, 2025]},$$
$$\mathrm{ACE}_{\mathcal{I}}(w) = \sup\{\mathrm{ACE}(h(w)) \mid h \in \mathcal{I}\},$$

where $\mathcal{I}$ is the set of all injective morphisms.

New "exponents"

Definition (Mapped exponents)

$$\begin{aligned}
\mathrm{E}_{\mathcal{I}}(u) &= \sup\{\mathrm{E}(h(u)) \mid h \in \mathcal{I}\}, \\
\mathrm{IE}_{\mathcal{I}}(u) &= \sup\{\mathrm{IE}(h(u)) \mid h \in \mathcal{I}\} \text{ [Saarela, 2025]}, \\
\mathrm{ACE}_{\mathcal{I}}(w) &= \sup\{\mathrm{ACE}(h(w)) \mid h \in \mathcal{I}\},
\end{aligned}$$

where $\mathcal{I}$ is the set of all injective morphisms.

We could also define $\mathrm{CE}_{\mathcal{I}}$, but that would be infinite for all infinite words by $h(a) = a^n$. (There might be interesting additional or alternative restrictions on morphisms such that CE could also be studied.)

Table of Contents

Example of an infinite $\mathrm{IE}_{\mathcal{I}}$ and $\mathrm{E}_{\mathcal{I}}$

Examples

Let $u = abaaba = (aba)^2$.
Let $h_k \in \mathcal{I}$ be morphisms defined by $h_k(a) = a$ and $h_k(b) = b(aab)^k$ for all $k \in \mathbb{N}$.
Then $h_k(u) = (a\ b(aab)^k\ a)^2 = ((aba)^{k+1})^2 = (aba)^{2k+2}$. Thus $\mathrm{IE}_{\mathcal{I}}(u) = \mathrm{E}_{\mathcal{I}}(u) = \infty$.
Let $w = abaab = ua^{-1}$. A theorem in the next slide tells that $\mathrm{IE}_{\mathcal{I}}(w) = 1$.
However, since $h_k(w)a = h_k(u) = (aba)^{2k+2}$, we have $h_k(w) = (aba)^{2k+2-\frac{1}{3}}$. Thus $\mathrm{E}_{\mathcal{I}}(w) = \infty$.

Finite $\mathrm{IE}_{\mathcal{I}}$

Theorem (J. Spehner, 1976)

Let $w \in \{a, b\}^*$ be a primitive ($\mathrm{IE}(w) = 1$) word. Then

$$\mathrm{IE}_{\mathcal{I}}(w) < \infty \iff \mathrm{IE}_{\mathcal{I}}(w) = 1 \iff |w|_a \geq 2 \text{ and } |w|_b \geq 2.$$

Theorem (A. Saarela, 2025)

Let $w \in \Sigma^*$ be a primitive word. Then

$$\mathrm{IE}_{\mathcal{I}}(w) < \infty \iff \mathrm{IE}_{\mathcal{I}}(w) < |w| \iff |w|_a \geq 2 \text{ for all } a \in \mathrm{alph}(w).$$

Low $\mathrm{E}_{\mathcal{I}}$

Short words often have infinite $\mathrm{E}_{\mathcal{I}}$. The following Theorem shows that $\mathrm{E}_{\mathcal{I}}(ababba) = 3$, and that there exists words with the $\mathrm{E}_{\mathcal{I}}$ being arbitrarily close to 1.

Theorem

For all $n \geq 2$ we have $\mathrm{E}_{\mathcal{I}}((ab)^n ba) = 1 + \frac{2}{n-1}$.

Low $\mathrm{E}_{\mathcal{I}}$

Short words often have infinite $\mathrm{E}_{\mathcal{I}}$. The following Theorem shows that $\mathrm{E}_{\mathcal{I}}(ababba) = 3$, and that there exists words with the $\mathrm{E}_{\mathcal{I}}$ being arbitrarily close to 1.

Theorem

For all $n \geq 2$ we have $\mathrm{E}_{\mathcal{I}}((ab)^n ba) = 1 + \frac{2}{n-1}$.

The only thing we need for this Theorem is the theorem of Fine and Wilf (or maybe trivial version of it).

Theorem (N. Fine and H. Wilf, 1965)

If a repetition of u and a repetition of v have a common prefix of length $|u| + |v| - \gcd(|u|, |v|)$, then u and v are integer powers of a common word.

Proof of low $\mathrm{E}_{\mathcal{I}}$ Theorem

First, $\mathrm{E}_{\mathcal{I}}((ab)^n ba) \leq 1 + \frac{2}{n-1}$.

Proof of low $\mathrm{E}_{\mathcal{I}}$ Theorem

First, $\mathrm{E}_{\mathcal{I}}((ab)^n ba) \leq 1 + \frac{2}{n-1}$.

- Let $u = (ab)^n ba$ and $h \in \mathcal{I}$ such that $h(u) = v^r$ for some primitive v. Now a repetition of $h(ab)$ and a repetition of v have a common prefix of length $|h(ab)^n|$.

Proof of low $\mathrm{E}_{\mathcal{I}}$ Theorem

First, $\mathrm{E}_{\mathcal{I}}((ab)^n ba) \leq 1 + \frac{2}{n-1}$.

- Let $u = (ab)^n ba$ and $h \in \mathcal{I}$ such that $h(u) = v^r$ for some primitive v. Now a repetition of $h(ab)$ and a repetition of v have a common prefix of length $|h(ab)^n|$.
- If this prefix is of long enough length ($\geq |h(ab)| + |v|$), then Theorem of Fine and Wilf states that words $h(ab)$ and v are integer powers of a common word. By primitivity of v, $h(ab) = v^m$.

Proof of low $\mathrm{E}_{\mathcal{I}}$ Theorem

First, $\mathrm{E}_{\mathcal{I}}((ab)^n ba) \leq 1 + \frac{2}{n-1}$.

- Let $u = (ab)^n ba$ and $h \in \mathcal{I}$ such that $h(u) = v^r$ for some primitive v. Now a repetition of $h(ab)$ and a repetition of v have a common prefix of length $|h(ab)^n|$.
- If this prefix is of long enough length ($\geq |h(ab)| + |v|$), then Theorem of Fine and Wilf states that words $h(ab)$ and v are integer powers of a common word. By primitivity of v, $h(ab) = v^m$.
- By the above equality and the assumptions, $h(u) = v^{nm} h(ba) = v^r$.

Proof of low $\mathrm{E}_{\mathcal{I}}$ Theorem

First, $\mathrm{E}_{\mathcal{I}}((ab)^n ba) \leq 1 + \frac{2}{n-1}$.

- Let $u = (ab)^n ba$ and $h \in \mathcal{I}$ such that $h(u) = v^r$ for some primitive v. Now a repetition of $h(ab)$ and a repetition of v have a common prefix of length $|h(ab)^n|$.
- If this prefix is of long enough length ($\geq |h(ab)| + |v|$), then Theorem of Fine and Wilf states that words $h(ab)$ and v are integer powers of a common word. By primitivity of v, $h(ab) = v^m$.
- By the above equality and the assumptions, $h(u) = v^{nm} h(ba) = v^r$.
- After canceling v^{nm}, $h(ba) = v^{r-mn} = v^q$ for some $q \in \mathbb{Q}$.

Proof of low $\mathrm{E}_{\mathcal{I}}$ Theorem

First, $\mathrm{E}_{\mathcal{I}}((ab)^n ba) \leq 1 + \frac{2}{n-1}$.

- Let $u = (ab)^n ba$ and $h \in \mathcal{I}$ such that $h(u) = v^r$ for some primitive v. Now a repetition of $h(ab)$ and a repetition of v have a common prefix of length $|h(ab)^n|$.
- If this prefix is of long enough length ($\geq |h(ab)| + |v|$), then Theorem of Fine and Wilf states that words $h(ab)$ and v are integer powers of a common word. By primitivity of v, $h(ab) = v^m$.
- By the above equality and the assumptions, $h(u) = v^{nm} h(ba) = v^r$.
- After canceling v^{nm}, $h(ba) = v^{r-mn} = v^q$ for some $q \in \mathbb{Q}$.
- Since $|h(ba)| = |h(ab)|$, $h(ba) = v^m = h(ab)$, a contradiction with $h \in \mathcal{I}$.

Proof of low $\mathrm{E}_{\mathcal{I}}$ Theorem

First, $\mathrm{E}_{\mathcal{I}}((ab)^n ba) \leq 1 + \frac{2}{n-1}$.

- Let $u = (ab)^n ba$ and $h \in \mathcal{I}$ such that $h(u) = v^r$ for some primitive v. Now a repetition of $h(ab)$ and a repetition of v have a common prefix of length $|h(ab)^n|$.
- If this prefix is of long enough length ($\geq |h(ab)| + |v|$), then Theorem of Fine and Wilf states that words $h(ab)$ and v are integer powers of a common word. By primitivity of v, $h(ab) = v^m$.
- By the above equality and the assumptions, $h(u) = v^{nm} h(ba) = v^r$.
- After canceling v^{nm}, $h(ba) = v^{r-mn} = v^q$ for some $q \in \mathbb{Q}$.
- Since $|h(ba)| = |h(ab)|$, $h(ba) = v^m = h(ab)$, a contradiction with $h \in \mathcal{I}$.
- To avoid this contradiction, we must have $|h(ab)^n| < |h(ab)| + |v|$, and thus $(n-1)|h(ab)| < |v|$. We then have

$$r = \frac{|h(u)|}{|v|} < \frac{(n+1)|h(ab)|}{(n-1)|h(ab)|} = 1 + \frac{2}{n-1}.$$

Proof of low $\mathrm{E}_{\mathcal{I}}$ Theorem

Next, $\mathrm{E}_{\mathcal{I}}((ab)^n ba) \geq 1 + \frac{2}{n-1}$.

- Let h_k be a morphism defined by $h_k(a) = (ab)^k a$ and $h_k(b) = ba$.
- It can then verified that $h_k(u) = [((ab)^k aba)^{n-1} ab]^{1+\frac{4k+4}{(2k+3)(n-1)+2}}$.
- Thus $\mathrm{E}_{\mathcal{I}}(u) \geq \lim_{k\to\infty} 1 + \frac{4k+4}{(n-1)(2k+3)+2} = 1 + \frac{2}{n-1}$.

Theorem

For all $n \geq 2$ we have $\mathrm{E}_{\mathcal{I}}((ab)^n ba) = 1 + \frac{2}{n-1}$.

Charaterizing words with infinite $\mathrm{E}_{\mathcal{I}}$

Motivated by Saarela's characterization of the words with $\mathrm{IE}_{\mathcal{I}} = \infty$, we wanted to find the language (over infinite alphabet)

$$L = \{w \mid \mathrm{E}_{\mathcal{I}}(w) = \infty\}.$$

However, it turns out that unlike in the case of $\mathrm{IE}_{\mathcal{I}}$, (we think) there is a need to separately consider different alphabet sizes.

Charaterizing words with infinite $\mathrm{E}_{\mathcal{I}}$

We proved that $\mathrm{E}_{\mathcal{I}}(w) = \infty$ implies that some letter in the word w must have a fixed factor between every two occurrences of it.

proof sketch.

- If $\exists h \in \mathcal{I}$ such that $h(w) = x^r$ for some $r > |w|$ and primitive x, then $\exists a \in \mathrm{alph}(w)$ such that $|h(a)| > |x|$.
- A conjugate of a primitive word x can be a factor of x^r in only obvious ways.
- This implies that each $h(a)$ has a fixed "place" within the repetition of x.
- So if u is in-between word for two occurrences of $h(a)$, then $h(a)u$ is a whole-number repetition of some (fixed) conjugate of x.
- In the preimage side, if there is two different words between consecutive a's, then we can construct words that contradict the injectivity of h.

Figure of the proof

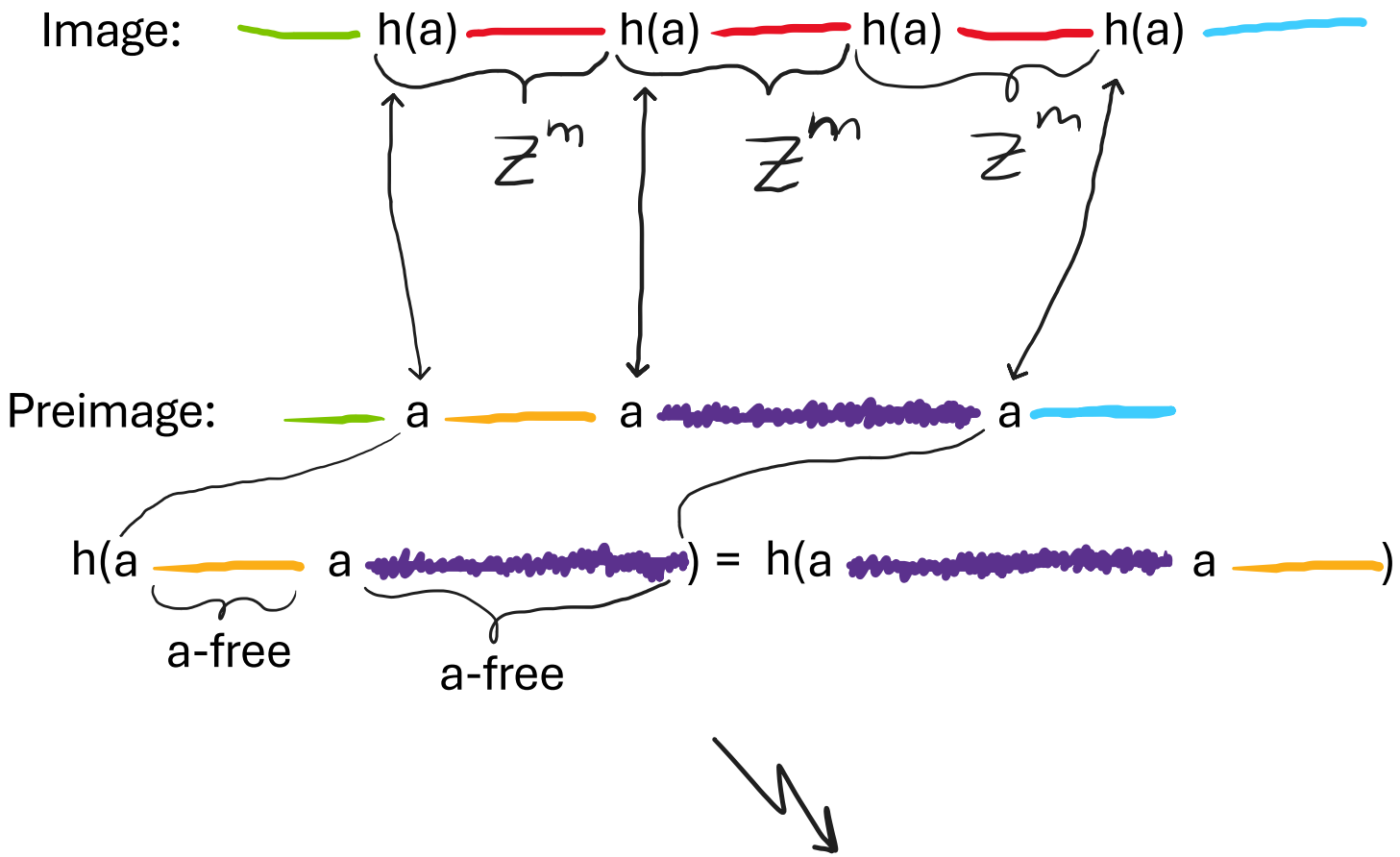

Figure: When $h(w) = x^n$ and $|h(a)| \geq |x|$, then yellow and purple factors must be the same.

Charaterizing words with infinite $\mathrm{E}_{\mathcal{I}}$

An easy case where we have a fixed factor between every occurrence of the letter a is a word of the type

$$uavuavua\cdots = (uav)^r$$

for some a-free words u, v and $r \geq 1$.
Define h_k for all $k \in \mathbb{N}$ by $h_k(a) = a(vua)^k$ and fixing all other letters.
Then it is easy to see that

$$h_k((uav)^r) = (uav)^R$$

for some $R > k$.
So all words of the type $(uav)^r$ have infinite $\mathrm{E}_{\mathcal{I}}$.

Charaterizing words with infinite $\mathrm{E}_{\mathcal{I}}$

We can extend previous easy case to words $w = w_1(aw_2)^k aw_3$ where w_1, w_2 and w_3 all **commute** and are a-free. Then let $h(a) = w_3 a w_1$ and fix all other letters. Then

$$\begin{aligned} h(w) &= w_1(w_3 a w_1 w_2)^k w_3 a w_1 w_3 \\ &= (w_1 w_3 a w_2)^k w_1 w_3 a w_1 w_3 \\ &= (w_1 w_3 a w_2)^{k+1} w_2^{-1} w_1 w_3 \\ &=^* (w_1 w_3 a w_2)^{k+1+\frac{|w_1 w_3|-|w_2|}{|w_1 w_3 a w_2|}} \end{aligned}$$

Since we can combine two injective morphisms to a single injective morphism, this case also gives infinite $\mathrm{E}_{\mathcal{I}}$ by the previous slide.
*This holds since $w_1, w_2, w_3 \in v^*$ for some word v.

Binary case

Over the binary alphabet these easy cases are the full story:

Theorem

$$L_2 = \{w \in \{a,b\}^* \mid \mathrm{E}_{\mathcal{I}}(w) = \infty\} = \bigcup_{k\in\mathbb{N}} b^*a(b^ka)^*b^* + a^*b(a^kb)^*a^*.$$

The shortest interesting binary word is $aabb$ (G. Fici, 2024). To map this word to arbitrary high powers with injective morphisms, we could first map $b \to baa$ and fix the letter a, obtaining $aabb \to aabaabaa = (aab)^{\frac{8}{3}}$. Then we map $b \to b(aab)^k$, fixing the letter a again.
We obtain $aabb \to (aab)^{\frac{8}{3}} \to (aab)^{\frac{8}{3}+2k}$.

Main theorem on finite words

Theorem

For a word $w \in \Sigma^+$, the following are equivalent:

1. $\mathrm{E}_{\mathcal{I}}(w) = \infty$.
2. $\mathrm{E}_{\mathcal{I}}(w) > |w|$.
3. *There exists an injective morphism $h \in \mathcal{I}$ and a letter $a \in \mathrm{alph}(w)$ such that $h(w) = x^r$ for some primitive $x \in \Gamma^*$, $r \in \mathbb{Q}$ and $|h(a)| \geq |x|$.*
4. *There exists an integer $k \geq 0$, a letter $a \in \Sigma$, words $w_1, w_2, w_3 \in (\Sigma \setminus \{a\})^*$, and an injective morphism h such that $w = w_1(aw_2)^k aw_3$, $h(w_1)$ and $h(w_2)$ are suffix-comparable, and $h(w_2)$ and $h(w_3)$ are prefix-comparable.*

The additional condition on fourth statement is necessary. The word *bbccabcbca* has finite $\mathrm{E}_{\mathcal{I}}$. Here $w_1 = bbcc$ and $w_2 = cbcb$ prevent the letter a to map to large word compared to primitive root of the image.

Note on the words of length 6

The shortest words with a finite $\mathrm{E}_{\mathcal{I}}$ are of length 6, all binary. Here six are shown. One obtains the complete list by flipping a's and b's.

w	$\mathrm{E}_{\mathcal{I}}(w)$
$aabbab$	3
$abaabb$	3
$ababba$	3
$abbaba$	3
$aababb$	2
$abbaab$	1.5

Table of Contents

Definition of $\mathrm{ACE}_{\mathcal{I}}$

$$\begin{aligned}\mathrm{ACE}_{\mathcal{I}}(w) &= \sup\{\mathrm{ACE}(h(w)) \mid h \in \mathcal{I}\} \\ &= \sup_{h\in\mathcal{I}} \big(\limsup_{n\to\infty} \{\mathrm{E}(f) \mid f \in \mathrm{Fact}_n(h(w))\}\big).\end{aligned}$$

It is a bit more difficult to get a feel for this definition compared to its finite counterparts, as any finite part of the infinite word tells nothing about $\mathrm{ACE}_{\mathcal{I}}$.

An example where ACE increases under injective morphism

Let $w = u_1 u_2 u_3 \cdots \in \{a, b\}^{\mathbb{N}}$ be a word with $\mathrm{ACE}(w) = 1$, factored so that $|u_i| = i$. Let $\sigma(a) = 0$ and $\sigma(b) = 1$ and

$$w' = \prod_{i=1}^{\infty} u_i \sigma(u_i) \overset{\text{for example}}{=} a0ab01aba010a \cdots .$$

It is not hard to believe that $\mathrm{ACE}(w') = 1$.
Let $h(a) = ac, h(b) = bc, h(0) = ca, h(1) = cb$. Then $h(ab01) = acbccacb = (acbcc)^{\frac{8}{5}}$, and in general

$$h(u_i \sigma(u_i)) = (h(u_i)c)^{\frac{4|u_i|}{2|u_i|+1}}.$$

So $\mathrm{ACE}_{\mathcal{I}}(w') \geq 2$.

Bound on $\mathrm{ACE}_{\mathcal{I}}$

We wanted to see if we can once again describe the words with $\mathrm{ACE}_{\mathcal{I}} = \infty$. The end result is pleasantly clean. However, the upper bound we ended up with is quite weak.

Theorem

For infinite word $w \in \Sigma^{\mathbb{N}}$,

$$\mathrm{ACE}_{\mathcal{I}}(w) = \infty \iff \mathrm{ACE}(w) = \infty.$$

If $\mathrm{ACE}(w) < \infty$, *then*

$$\mathrm{ACE}_{\mathcal{I}}(w) \leq |\Sigma| + 1 + |\Sigma|(|\Sigma| + 1)(\mathrm{ACE}(w) + 1).$$

Binary case

Theorem (L. Dvořáková, P. Ochem and D. Opočenská,2024)

Let h be an injective morphism, u an infinite word with **uniform letter frequencies** *and L a natural number. If all factors f of $h(u)$ with $|f| \geq L$ are* **synchronizing words** *of h, then $\mathrm{ACE}(h(u)) = \mathrm{ACE}(u)$.*

Theorem

Let h be an injective morphism, u an infinite word and L a natural number. If all factors f of $h(u)$ with $|f| \geq L$ are synchronizing words of h and $\mathrm{ACE}(u) \in \mathbb{R}$, then $\mathrm{ACE}(h(u)) \leq \lceil \mathrm{ACE}(u) \rceil$.
Moreover, if u is over the binary alphabet and $\mathrm{ACE}(u) \notin \mathbb{N}$, then there exists absolute constant $\lambda_u > 0$ such that for all h and L pairs, $\mathrm{ACE}(h(u)) \leq \lceil \mathrm{ACE}(u) \rceil - \lambda_u$.

Figure of repetitions with synchronizing words

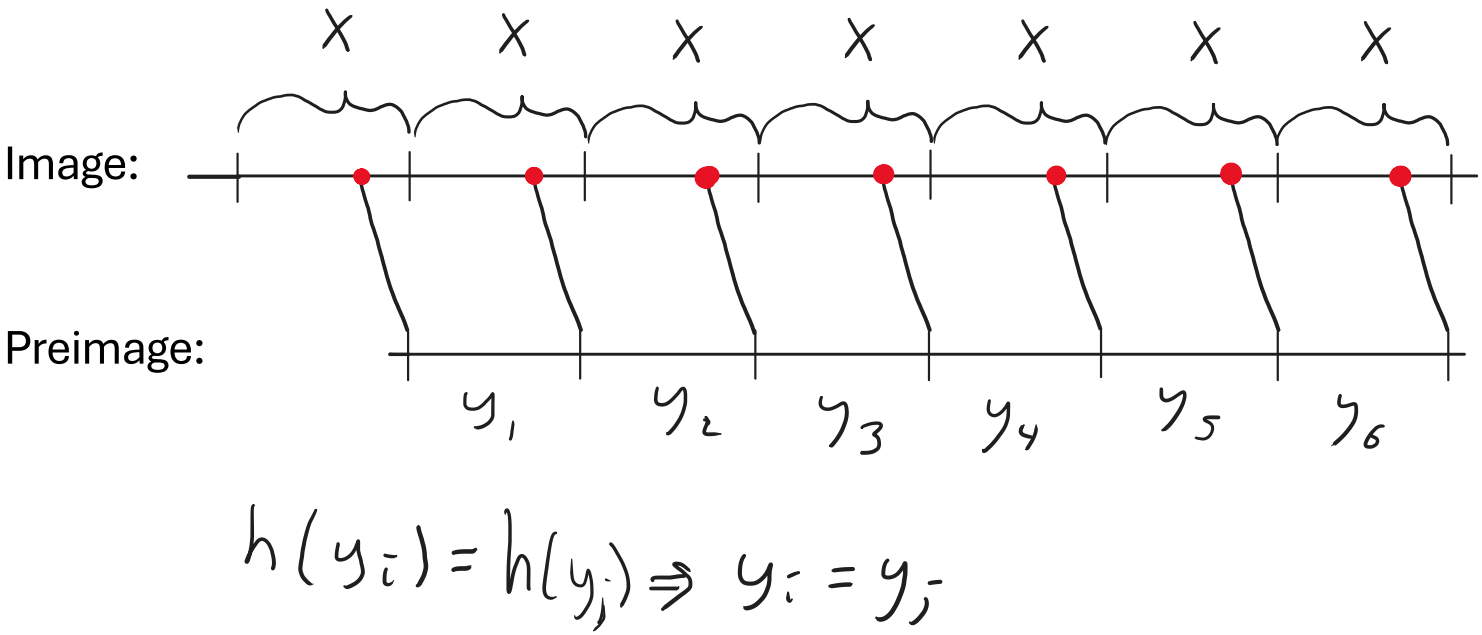

Figure: If x is a synchronizing word for h, then we know some letter boundaries the preimage has just by looking the image. By injectivity, repetitions of x then imply repetitions in the preimage. The actual situation is a bit more complicated, here only the basic idea is illustrated.

Binary synchronizing words

Theorem

Let h be an injective morphism and w an infinite binary word. If there exists infinite list $f_1, f_2, f_3, \ldots$ of factors of $h(w)$ that are not synchronizing words of h and $\lim_{i\to\infty} |f_i| = \infty$, then $\mathrm{ACE}(h(w)) = \mathrm{ACE}(w) = \infty$.

proof sketch.

$f_1 = ..$

$f_2 = ...$

$f_3 = ...$

$f_4 = ...$

$f_5 = ...$

.

.

By compactness, aligned like in the left, the factors f_i have a two-sided infinite limit word.
By assumptions, this limit word has two disjoint preimages by the morphism h.
In the case of binary morphisms, this can only happen if this limit word is periodic by results of Karhumäki, Maňuch and Plandowski (2003) on two-sided infinite words and defect effect.

The binary result

Theorem

Let w be an infinite binary word with $\mathrm{ACE}(w) \in \mathbb{R}$. If $\mathrm{ACE}(w) \in \mathbb{N}$ or w has uniform letter frequencies, then $\mathrm{ACE}_{\mathcal{I}}(w) = \mathrm{ACE}(w)$. Otherwise, $\mathrm{ACE}_{\mathcal{I}}(w) < \lceil \mathrm{ACE}(w) \rceil$.

Proof.

By $\mathrm{ACE}(w) \in \mathbb{R}$ there exists some length after which all factors of $h(w)$ are synchronizing. Then if the word w has uniform letter frequencies, we apply the theorem by Dvořáková, Ochem and Opočenská. If not, we apply our modification of it. □

We also have a construction that shows this upper bound to be exact.

Conclusion

Finite words:

- There exists words with $\mathrm{E}_{\mathcal{I}}$ being arbitrary close to one.
- All words with $\mathrm{E}_{\mathcal{I}} = \infty$ have a factorization $w_1 a(w_2 a)^k w_3$ for some letter a and a-free words w_1, w_2 and w_3. Words with $\mathrm{IE}_{\mathcal{I}} = \infty$ have $k = 0$.
- This factorization characterizes the binary and the $\mathrm{IE}_{\mathcal{I}}$ cases completely.

Infinite words:

- $\mathrm{ACE}_{\mathcal{I}}$ is finite if and only if ACE is finite.
- For a binary word $w \in \{a, b\}^{\mathbb{N}}$, $\mathrm{ACE}_{\mathcal{I}}(w) \leq \lceil \mathrm{ACE}(w) \rceil$ where equality happens only if $\mathrm{ACE}(w) \in \mathbb{N}$. This result is optimal.

References 1/2

Thank you for listening! Our papers:

- E. Foster, A. Saarela, A. Vanhatalo, 2025. Mapped Exponent and Asymptotic Critical Exponent of Words, DLT 2025.
- A. Saarela, 2025. Mapping words to powers by morphisms, Preprint.

Titles of the papers directly used in this talk:

- J. Beck, 1984. An application of Lovász local lemma: there exists an infinite 01-sequence containing no near identical intervals.
- J. Cassaigne, 2008. On extremal properties of the Fibonacci word.
- N. Fine and H. Wilf, 1965. Uniqueness theorems for periodic functions
- L. Dvořáková, P. Ochem, and D. Opočenská, 2024. Critical Exponent of Binary Words with Few Distinct Palindromes.
- J. Karhumäki, J. Maňuch, and W. Plandowski, 2003. A defect theorem for bi-infinite words.

References 2/2

Titles of the rest of the mentioned papers:

- R. Entringer, D. Jackson, and J. Schatz, 1974. On nonrepetitive sequences.
- J. Spehner, 1976. Quelques problémes d'extension, de conjugaison et de présentation des sous-monoïdes d'un monoïde libre