# ERC workshop on point processes and related topics

5-11 May 2016, Carry-le-Rouet, France

The workshop was devoted to the interactions between random matrix theory, determinantal and pfaffian point processes, domino tilings and plane partitions and actions of the infinite classical groups. We've achieved deeper insight and progress in research on the following topics:

- pfaffian Schur processes in the context of tilings;
- gas phase for the domino tiling models;
- description of the conditional measures for the determinantal point processes;
- 2-dimensional point processes, polyanalytic ensembles;
- asymptotics a Toeplitz determinants and it's applications in the theory of random matrices;
- ergodic measures for the actions of infinite classic groups in p-adic case, and orthogonal and symplectic case.

The abstracts of the talks are presented below.

**Dan Betea. A zoo of limits in a zoo of Schur processes**

We provide some asymptotics of certain determinantal or pfaffian processes called Schur processes, originally introduced by Okounkov and Reshetikhin. Special cases include tilings like plane partitions, pyramid partitions, the Aztec diamond and plane overpartitions as well as specific types of last passage percolation. Limits include various longest increasing subsequence problems. We will be concerned with limit shape phenomena and various limit kernels like the incomplete beta and its generalizations, GUE corners, Airy, cusp Airy, Pearcey, and possibly others. If time permits we will discuss the periodic Schur process and the periodic shifted Schur process which put the Schur (or shifted Schur) process on a cylinder.

**Alexander Bufetov. Conditional measures of determinantal point processes**

For a class of one-dimensional determinantal point processes including those induced by orthogonal projections with integrable kernels satisfying a growth condition, it is proved that their conditional measures, with respect to the configuration in the complement of a compact interval, are orthogonal polynomial ensembles with explicitly found weights. Examples include the sine-process and the process with the Bessel kernel. The argument uses the quasi-invariance of our point processes under the group of piecewise isometries of $\mathbb{R}$.

**Sunil Chhita. The two-periodic Aztec diamond**

Random domino tilings of the Aztec diamond shape exhibit interesting features and statistical properties related to random matrix theory. As a statistical mechanical model it can be thought of as a dimer model or as a certain random surface. We consider the Aztec diamond with a two-periodic weighting which exhibits all three possible phases that occur in these types of models, often referred to as solid, liquid and gas. In this talk, we introduce the model and focus on the behavior at the liquid-gas boundary. This is based on joint works with Vincent Beffara (Grenoble), Kurt Johansson (Stockholm) and Benjamin Young (Oregon).

**Haakan Hedenmalm. Coulomb gas ensembles and Laplacian growth**

We consider weight functions $Q : \mathbb{C} \to \mathbb{R}$ that are locally in a suitable Sobolev space, and impose a logarithmic growth condition from below. We use $Q$ as a confining potential in the model of one-component plasma (2-dimensional Coulomb gas), and study the configuration of the electron cloud as the number $n$ of electrons tends to infinity, while the confining potential is rescaled: we use $mQ$ in place of $Q$ and let $m$ tend to infinity as well. We show that if $m, n$ tend to infinity in a proportional fashion, with $n/m \to t$ , where $0 < t < +\infty$ is fixed, then the electrons accumulate on a compact set $S_t$, which we call the *droplet*. The set $S_t$ can be obtained as the coincidence set of an obstacle problem, if we remove a small set (the shallow points). Moreover, on the droplet $S_t$, the density of electrons is asymptotically $\Delta Q$. The growth of the droplets $S_t$ as $t$ increases is known as Laplacian growth. It is well-known that Laplacian growth is unstable. To analyze this feature, we introduce the notion of a *local droplet*, which involves removing part of the obstacle away from the set $S_t$. The local droplets are no longer uniquely determined by the time parameter $t$, but at least they may be partially ordered. We show that the growth of the local droplets may be terminated in a maximal local droplet, or by the droplets' growing to infinity in some direction ("fingering").

**Igor Krasovsky. 1. Random matrices and Toeplitz determinants**

We discuss some basic facts from the theory of random matrices using the examples of the Gaussian and Circular Unitary Ensembles. We mention the relation to Hankel, Toeplitz and special Fredholm determinants and their asymptotics.

**Igor Krasovsky. 2. Toeplitz determinants with merging singularities**

We describe the transition in the asymptotic behaviour of a Toeplitz determinant whose symbol has 2 distinct singularities as they merge together along the unit circle. We mention a connection to the moments of the characteristic polynomial of random matrices. Based on a joint work with Tom Claeys.

**Pavel Nikitin. Invariant ergodic measures for the actions of infinite classic groups on the spaces of infinite matrices**

We plan to remind VershikOlshanski ergodic method and the spectral decomposition of invariant measures, and then apply this methods for the actions of the infinite unitary group by conjugation on the space of infinite hermitian matrices (work of A.Vershik and G.Olshanski) and for the corresponding actions of the infinite orthogonal group $O(\infty)$ and the infinite symplectic group $Sp(\infty)$ on the spaces of infinite real antisymmetric and infinite quaternionic antihermitian matrices.

**Yanqi Qiu**

Let $F$ be a non-discrete non-Archimedean locally compact field and $O_F$ be the ring of integers in $F$. I will talk about the classification of ergodic probability measures on the space $\mathrm{Mat}(\mathbb{N}, F)$ of infinite matrices with respect to the natural action of the group $\mathrm{GL}(\infty, O_F) \times \mathrm{GL}(\infty, O_F)$. This talk is based on a joint work with Alexander Bufetov.