Polynomial de Rham sequences of arbitrary degree on polyhedral meshes

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In the context of standard differential operators, the de Rham sequence

$$\mathbb{R} \xrightarrow{i_\Omega} H^1(\Omega) \xrightarrow{\mathbf{grad}} \boldsymbol{H}(\mathbf{curl};\Omega) \xrightarrow{\mathbf{curl}} \boldsymbol{H}(\mathrm{div};\Omega) \xrightarrow{\mathrm{div}} L^2(\Omega) \xrightarrow{0} \{0\}$$

has *exactness* properties (for topologically trivial domain Ω): the image of an operator in the sequence is exactly the kernel of the next one. These properties are crucial to establish the well-posedness of certain partial differential equations, such as the Stokes/Navier–Stokes equations or equations involving magnetic fields. Designing a discrete version of the de Rham sequence enables to write schemes for these models, that are automatically well-posed and stable.

In this talk we will present such a discrete sequence. Its main features are:

1. It is applicable on meshes made of general polyhedras,
2. It is entirely based on explicit polynomial spaces (and thus fully implementable), and of arbitrary degree of accuracy,
3. The locations of the degrees of freedom naturally correspond to the physics represented by the differential operators,
4. It provides explicit potential reconstructions and L^2 inner products, that can be directly used to write discrete versions of weak formulations of PDEs.

We will detail the construction of the sequence, and discuss its properties: exactness, of course, but also Poincaré inequalities and consistency (both primal and adjoint). As an illustration we will show how this discrete de Rham sequence can be used to discretise a magnetostatics problem written in mixed form, and provide numerical results that illustrate the convergence properties of the resulting scheme.

This presentation is based on joint works with Daniele A. Di Pietro (Univ. Montpellier, France) and Francesca Rapetti (Univ. Nice, France).