

# Abstract

In the framework of the probabilistic approach to quantum field theories, in dimension 2 and in the Euclidean setting, we are interested in models exhibiting certain invariance properties under conformal transformations.

Toda conformal field theories (CFTs) are models in which the underlying field takes values in a vector space and generalize Liouville theory. Toda models are expected to exhibit, in addition to conformal invariance, a higher level of symmetry encoded by  $W$ -algebras. In this thesis, we focus on the case where the field is defined on a (compact, connected) Riemann surface with boundary.

A first contribution of this thesis is the construction of Toda models on hyperbolic surfaces, with or without boundary (generalizing the construction by Cerclé-Rhodes-Vargas on the sphere), using probabilistic tools. The presence of a boundary gives rise to new phenomena compared to Liouville CFT: different boundary conditions are possible for the underlying field, depending on the structure of the automorphism group of the target Lie algebra. We explore these different constructions and identify, in various cases, the symmetry algebra of the proposed models, in the sense of vertex algebras.

In a second part, we specialize the study to the simplest Toda CFT after Liouville, that is, when the field takes values in the algebra  $\mathfrak{sl}_3\mathbb{C}$ . We study this model on the complex half-plane. In this context, we establish the presence of enhanced symmetries in the form of boundary Ward identities, which was previously unknown in Physics. This result relies on the rigorous definition of descendants associated with the  $W$ -algebra on the boundary, using a regularization procedure and probabilistic tools. We then study the existence of singular vectors in the model and discover the presence of nonzero singular vectors on the boundary up to level 3, which is also a new phenomenon compared to the existing literature. These properties allow us to derive new hypergeometric differential equations of BPZ type for certain correlation functions of the theory, namely the bulk/boundary correlator and the boundary three-point function.

These results pave the way for the computation of these correlation functions, a key step in the implementation of the conformal bootstrap for Toda theories. They also highlight new phenomena in the study of two-dimensional conformal field theories in the presence of a boundary.

**Keywords:** conformal field theories, Toda theories, Riemann surfaces with boundary, Gaussian free field, Gaussian multiplicative chaos,  $W$ -algebras.