INSTITUT de MATHÉMATIQUES de MARSEILLE

Internship in image processing (4-6 months)

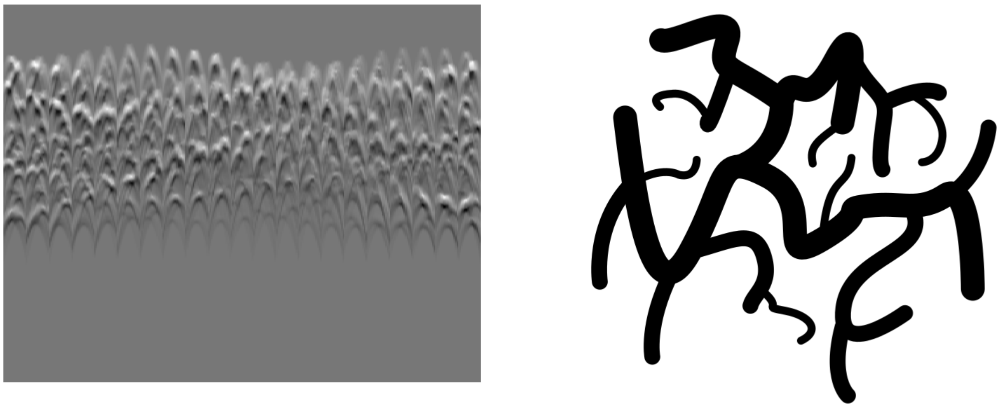
Photoacoustic tomographic image reconstruction

Year 2021-22

1 Context

Photoacoustic tomography (PAT) is a recent multi-wave image modality that shows great potential in biological and clinical research. This modality combines an optical excitation (laser) of the tissue with an ultrasound detection allowing to generate high-resolution images of optical absorption at large depths.

The optical absorption in biological tissues can be due to endogenous molecules such as hemoglobin or melanin, and to injected contrast agents. This absorption results in a local increase of the temperature in the tissue, creating a small pressure increase which in turn generates an acoustic wave. In 3D photoacoustic tomography (PAT), the ultrasound wavefield is measured, as an electrical signal, by several sensors distributed on a surface surrounding the tissue. The reconstruction of the image is then performed through the resolution of an inverse problem from the observed (measured) signals (see Fig. 1). The PAT modality is therefore intricate with its numerical reconstruction method and as such is an instance of computational imaging systems that have transformed modern imaging.



(a) (b)

Figure 1: (a) Image of measured signals (time vs detector position) and (b) the corresponding numerical vessel phantom

1.1 Physical modelling

Physically, the PAT can be modelled with two different phenomena:

The wave propagation in the tissue The initial pressure $p_0 : \mathbb{R}^3 \to \mathbb{R}$ generated by the optical absorption and to be recovered in the image is mapped to the whole pressure field at all times $p : \mathbb{R}^3 \times [0, T] \to \mathbb{R}$ by a linear operator $\mathcal{H} : p_0 \mapsto p$. This linear operator represents the wave equation: a second order linear partial differential equation (PDE). Once discretized, the matrix $\mathcal{H}$ is of size $(N^3 \times n_T) \times N^3$ where N^3 is the size of the grid discretizing the tissue domain and n_T is the number of time samples. In practice this matrix is huge and cannot be stored.

The measurements In PAT the pressure field p is measured by n sensors outside the tissue. Each sensor is associated to a 2D surface $S_i \subset \mathbb{R}^3$ that records the pressure. The associated measurement operator $\mathcal{M}_i$ maps the pressure p to the signal $g_i(t) = \int_{S_i} p(x,t)dS_i(x)$. The global measurement operator $\mathcal{M} = (\mathcal{M}_i)_{i=1}^n$ is thus the concatenation of all measurements $g = (g_i)_{i=1}^n$.

Overall, the physical modeling leads to the following linear inverse problem:

$$g = \mathcal{A}p_0 \tag{1}$$

with $\mathcal{A} = \mathcal{M}\mathcal{H}$ and where g are the observed signals and p_0 the initial pressure to be recovered. Once discretized, the matrix $\mathcal{A}$ is of size $(nn_T) \times N^3$. Note that the optimization algorithms implemented to solve (1) require the iterative application of $\mathcal{A}$ and its adjoint $\mathcal{A}^*$. Great care must be taken when implementing these matrix-vector products as they are critical for the efficiency of the reconstruction.

1.2 Numerical issues

Two issues make the resolution of this inverse problem difficult. First, the computation of $\mathcal{H}$ through wave propagation simulations is numerically intensive (3D + time). Second, the adjoint $\mathcal{A}^*$ is often implemented through back-propagation which has a simple expression when the sensors are isotropic and reduced to points. This however, requires to finely discretize the surfaces of each sensor, say with m points (that has to be chosen such that $m \propto N^2$ points), leading to a matrix $\mathcal{A}$ with m more non-zero coefficients. This prevents the reconstruction of the image p_0 in decent times leading researchers to resort to approximate sensor geometries which degrades the quality of reconstructed images.

2 Objectives

In preliminary works, we investigated an original approach, appearing to be new in the PAT community, that allows to fastly reconstruct p_0 without compromise on its quality. This approach is based on a fine implementation of the matrix-vector products with $\mathcal{A}$ and $\mathcal{A}^*$ in the optimization algorithms. With this approach, one iteration of the algorithm now costs $O(nN^3)$ operations.

The objectives of the internship are to make this proof of concept a universal method that can be used routinely in laboratories. We have, for instance, identified several needs:

- Although sensors are fully characterized in our method, they have an electrical impulse response that is crucial to model and incorporate in our method.
- The current implementation is made for images in 2D albeit real images are 3D. In order to scale the code in this setting, we need to implement the algorithms on massively parallel architectures such as graphics processing units (GPU). A speed-up of a factor $\times 60$ can be expected.
- The current implementation is based on fast Fourier transforms (FFT) that are extremely well implemented on any scientific computing library making it universal. However, the Fourier transform implicitly assumes periodic boundary conditions which forced us to significantly extend the domain to avoid the periodic waves to pollute the signals. A possible solution would be to implement a Perfectly Matched Layer which might lead to significant speed-ups ($\simeq \times 30$) in 3D.
- We believe that the matrix vector-products in our method implemented in $O(nN^3)$ which depends linearly in the number of sensors can be further reduce to $O(rN^3)$ with $r \ll n$ using dimensionality reduction techniques. This would have a dramatic impact on the speed of reconstruction algorithms. A speed-up of a factor $\times 10$ can be expected.
- If time allows, we might also investigate the co-conception of the PAT device. This means to adequately design the sensor locations to optimize the quality of the reconstructed images. This would allow to reduce the number of measurements needed and speed up the acquisition of the signals which is important in research and clinical applications.

Overall, the previous ideas might lead to a speed-up of order 10^4 which would make the algorithms efficient enough to reconstruct high-quality images without any compromise on their quality. This internship could have a great impact on the PAT community.

The candidate will be trained and could develop skills in optimization, image processing, high performance computing and approximation theory. These competences are actively being in demand in the industry and the academic research.

Internship information

The internship takes part in the project COCON3D which received a funding from the France Life Imaging program (`https://www.francelifeimaging.fr/`).

The training period will take place in the Signal and Image team I2M (`https://www.i2m.univ-amu.fr/Equipe-Signal-et-Image-SI`) of the Institut de Mathématiques de Marseille (`https://www.i2m.univ-amu.fr`), which is a joint research center between Aix-Marseille University, Ecole Centrale Marseille and CNRS (Centre National de la Recherche Scientifique). The Signal and Image team at I2M is easily accessible from downtown Marseille by public transportation (Metro Line 1 in the direction of La Rose until the last station La Rose. Then Bus B3B direction Technopôle de Château Gombert until the Technopôle Polytech Marseille stop).

As the interdisciplinary project is linked to the Laboratoire d'Imagerie Biomédicale (LIB) of Sorbonne Université, videoconference meetings with the LIB as well as meeting in Paris will also be organized with the student.

The internship might lead to a PhD thesis.

- Candidate profile: master of science with strong skills in statistics, signal processing, machine learning, or optimization. Languages: Python/Matlab.
- Duration: 4 to 6 months
- Location: Signal and Image team I2M, Institut de Mathématiques de Marseille (`https://www.i2m.univ-amu.fr`)
- Supervision: Paul Escande (`paul.escande@univ-amu.fr`), Caroline Chaux (`caroline.chaux@univ-amu.fr`).
- Stipend: about 600 euros per month.
- Application: send resume, master grades and motivation letter to supervisors; please name documents as **LASTNAME_Firstname-Resume.***, **LASTNAME_Firstname-Grades.*** or **LASTNAME_Firstname-Letter.***

References

[1] Rosenthal, Ntziachristos, Razansky, Acoustic Inversion in Optoacoustic Tomography: A Review. Curr. Med. Imaging Rev. 9, 318 (2013)

[2] X. L. Dean-Ben, V. Ntziachristos and D. Razansky, "Acceleration of Optoacoustic Model-Based Reconstruction Using Angular Image Discretization," in IEEE Transactions on Medical Imaging, vol. 31, no. 5, pp. 1154-1162 (2012)

[3] E. Candès, L. Demanet, L. Ying, Fast Computation of Fourier Integral Operators, SIAM Journal on Scientific Computing, 29:6, 2464-2493 (2007)

[4] P. L. Combettes, J.-C. Pesquet, Proximal Splitting Methods in Signal Processing, in Fixed-Point Algorithms for Inverse Problems in Science and Engineering, Springer New York, 185-212 (2011)

[5] N. Komodakis and J. Pesquet, "Playing with Duality: An overview of recent primal-dual approaches for solving large-scale optimization problems," in IEEE Signal Processing Magazine, vol. 32, no. 6, pp. 31-54 (2015)